Rayleigh-Taylor Unstable Flames

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Type Ia Supernovae

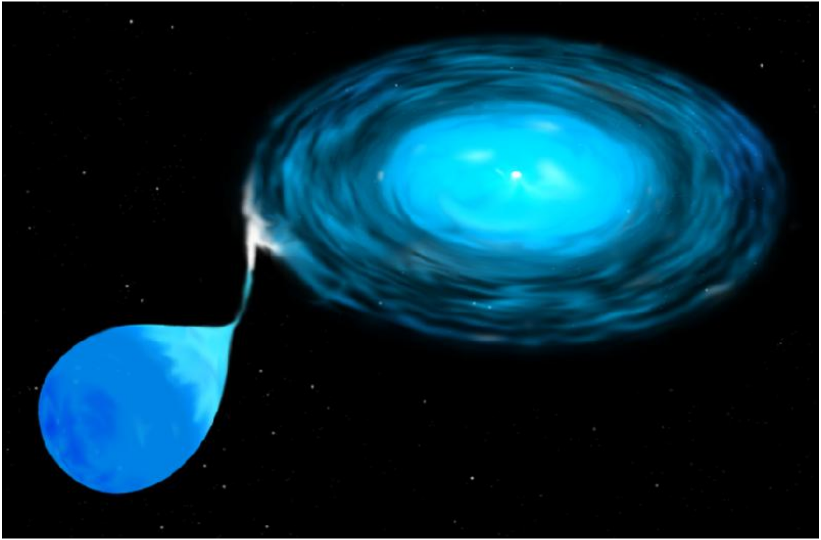


Image: NASA

Deflagration to Detonation Transition (DDT)

In order to get the correct explosion properties the SN Ia flame must become supersonic.

- Deflagration: A subsonic flame
- Detonation: A supersonic flame with an associated shock wave

Two ways for this to occur:

- Zeldovich Gradient Mechanism
- Flame exceeds the sound speed because fluid instabilities (Rayleigh-Taylor instability) increase the surface area of the flame

Rayleigh-Taylor Instability

The source of wrinkling: the Rayleigh-Taylor instability

- The fuel is more dense than the ash
- The flame propages upward against the direction of gravity

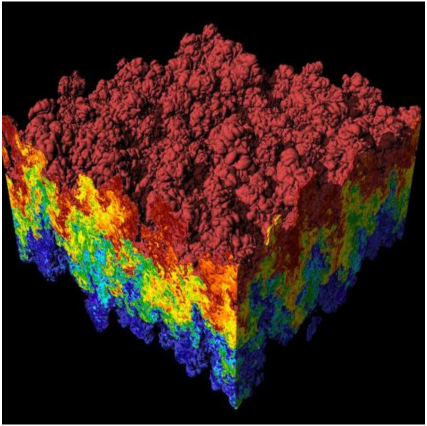


Image: LLNL

Some important questions:

How do flows generated by the flame interact with the flame front?

- What causes changes in the surface area of the flame?
- At what G does the Rayleigh-Taylor instability become important?
- How much can Rayleigh-Taylor driven turbulence wrinkle the flame front?

Difficulties for Simulations

1. **Current Supercomputers are unable to resolve the flame and the whole white dwarf at the same time.**
   - Size of White Dwarf$= 7 * 10^8$cm, Width of Flame$= .01 - 1$ cm
   - Subgrid Models for flame behavior are necessary

What kind of subgrid model is actually appropriate?

- Flame Speed = Rayleigh-Taylor speed
- Flame Speed determined by Kolmogorov Turbulence

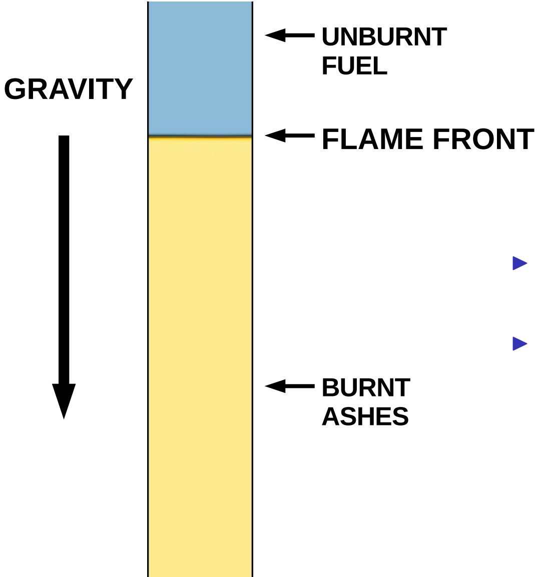
2. **Full Reaction Networks are very stiff and difficult to integrate**

Study Flames Directly

Avoid these problems and study a very simplified case:

1. A flame in a rectangular computational domain
2. Constant gravity
3. Use a simple, model reaction

Flame in a Gravitational Field



- The flame is Rayleigh-Taylor unstable.
- Code: Nek5000 (P. Fischer, ANL)

Important Parameters

Non-Dimensional Gravity:

$$G = g\left(\frac{\triangle\rho}{\rho_o}\right)\frac{\delta}{s_o^2}$$

Prandtl Number:

$$Pr = \frac{\nu}{\kappa} = 1$$

Non-Dimensional Box Size:

$$L = \frac{l}{\delta} = 128$$

What happens when gravity is increased?

Transition-to-turbulence type problem

- Look for low-dimensional dynamical systems and simple bifurcations when G is small
- Consider the effects of turbulence when G is large

Dynamical System

A **dynamical system** consists of a state space plus a rule for time evolution in that space.

$x = \Theta$
$y = \dot{\Theta}$

$\dot{y} = -\frac{g}{L} sin(x)$
$\dot{x} = y$

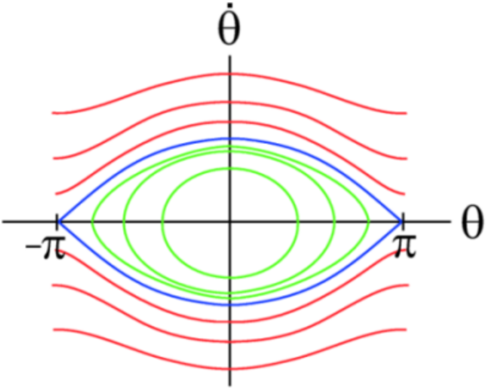


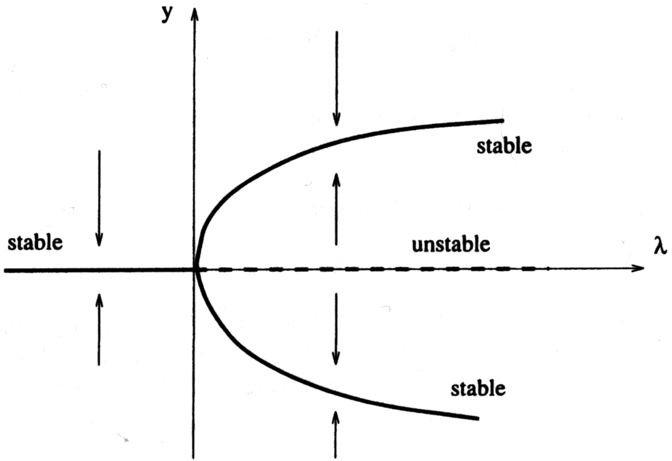
Image: S. Shadden

Bifurcations

A **bifurcation** is a change in a system as a **control parameter** is increased. Example:

$$\dot{y} = \lambda y - y^3$$

λ is the **control parameter**



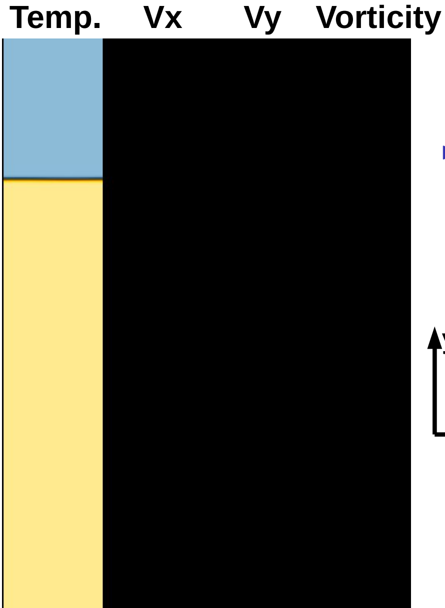
Low G Systems

General Strategy:

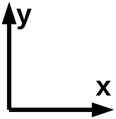
- Pick an observable to model (examples: velocity at a point, flame speed)
- Treat the observable as part of an underlying low-dimensional dynamical system.

Spatial behaviors can be understood temporally

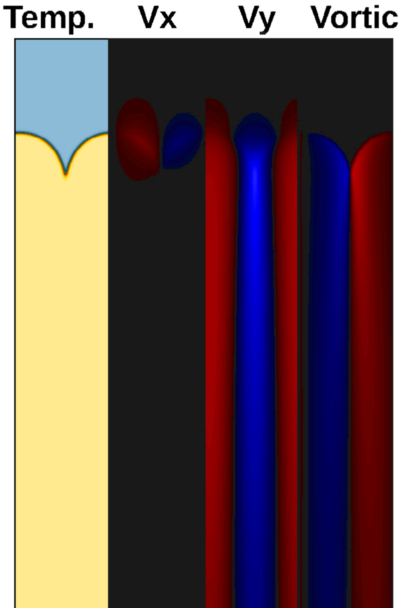
Flame: Fixed Point Example ($G = 0.001$)



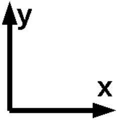
- The flame moves upward at a constant speed.



Cusped Flame Front, Stable Rolls ($G = 0.17$)

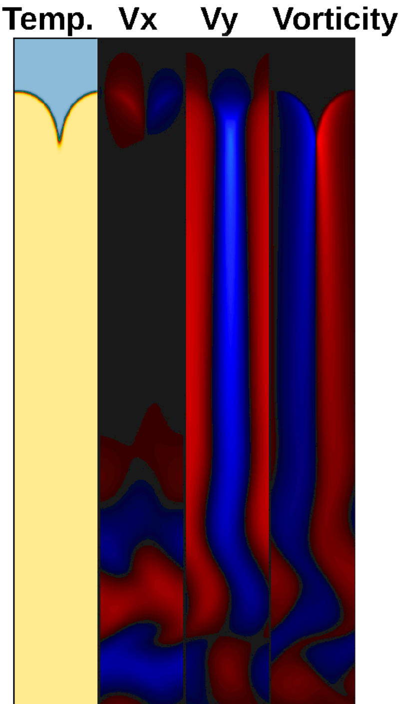


- Flame front becomes **cusped**.
- **Stable rolls** attach to the flame front
- Flame speed is still constant



Unstable Rolls ($G = 0.24$)

- **Shear instability** destabilizes the rolls
- **Vortex Shedding** begins!
- Flame speed is still constant



Is there a low-dimensional model that describes the vortex shedding?

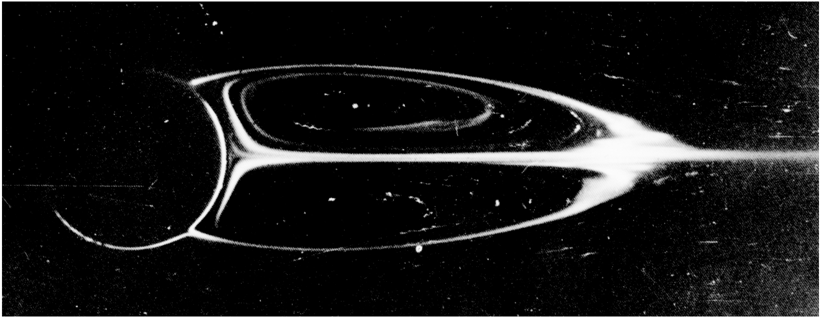


Image: Van Dyke

- Vortex shedding creates periodic behavior: **Hopf bifurcation**
- Similar system: flow past a cylinder (von Karman vortex street)
- Cylinder: Shedding at near critical Re governed by the Landau equation (Strykowski and Sreenivasan 1986,1987,1990)

Landau Equation: Governs shedding behind the flame

The Landau equation is satisfied in the vortex forming region behind the flame.

$$\frac{dA}{dt} = aA - \frac{1}{2}c|A|^2 A$$

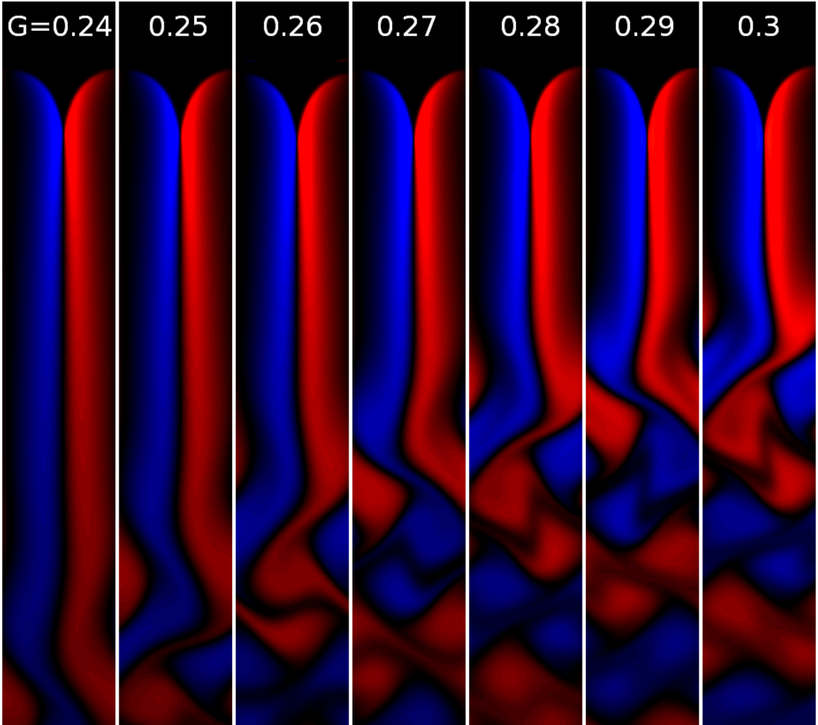
- In this case, choose V_x as A
- The beginning of periodic motion is a Hopf bifurcation.
- $G_{cr} \approx .22$

Vortex shedding is can be modelled by a time-dependent process with a secondary spatial dependence.

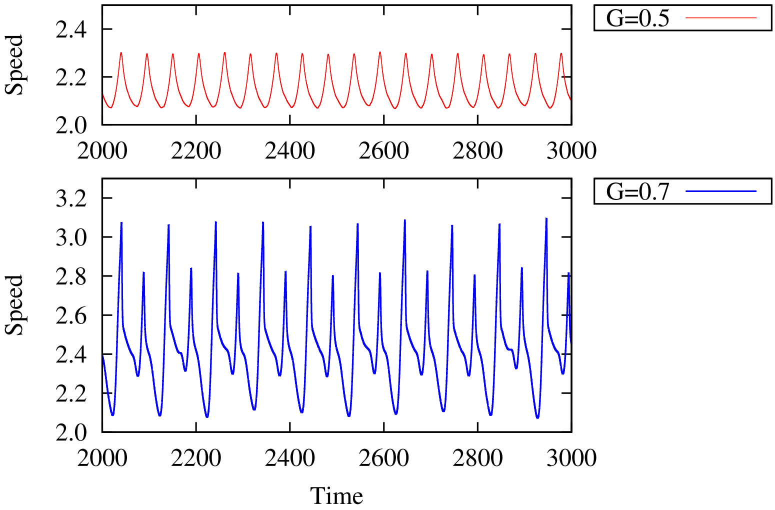
Time Series Analysis of the Flame Speed

Observe the flame speed to learn about the underlying "dynamical system"

- When vortex shedding moves close enough to the flame front, the flame speed begins to oscillate. ($G \approx 0.3$)
- Period Doubling: Left/Right Symmetry Breaks ($G \approx 0.6$)
- Torus Bifurcation: Cusp Breaks ($G \approx 0.85$)

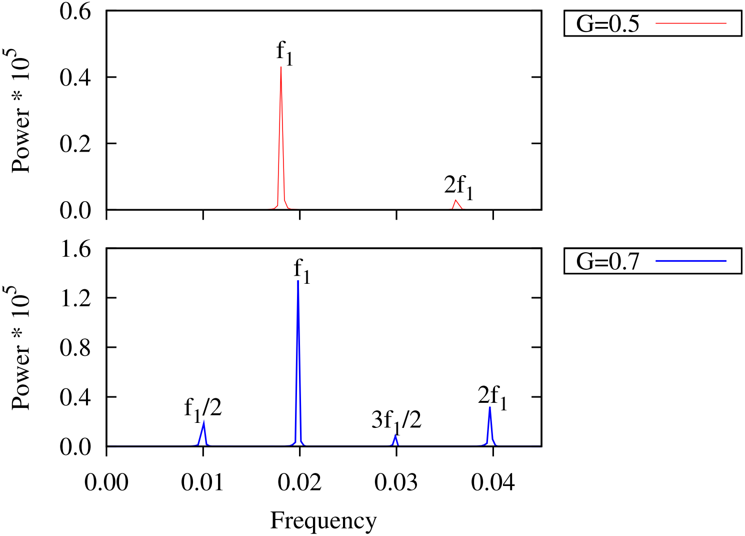

G=0.24
0.25
0.26
0.27
0.28
0.29
0.3

Period Doubling

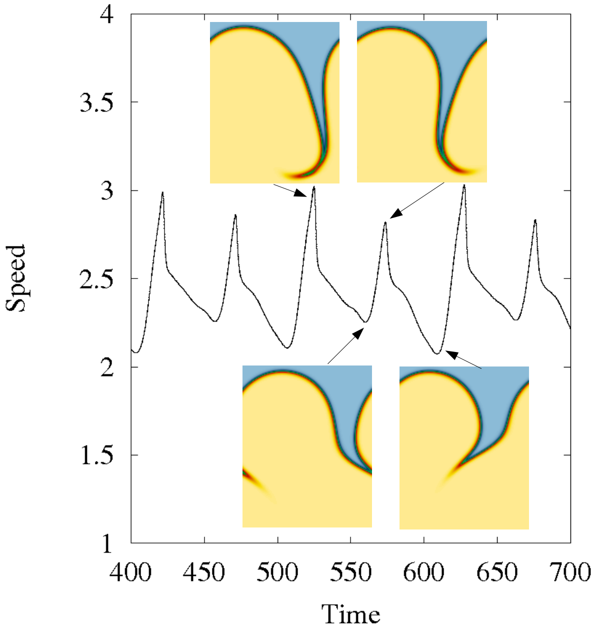


- Between $G = 0.5$ and $G = 0.7$ a period doubling occurs.

Period Doubling: Power Spectra

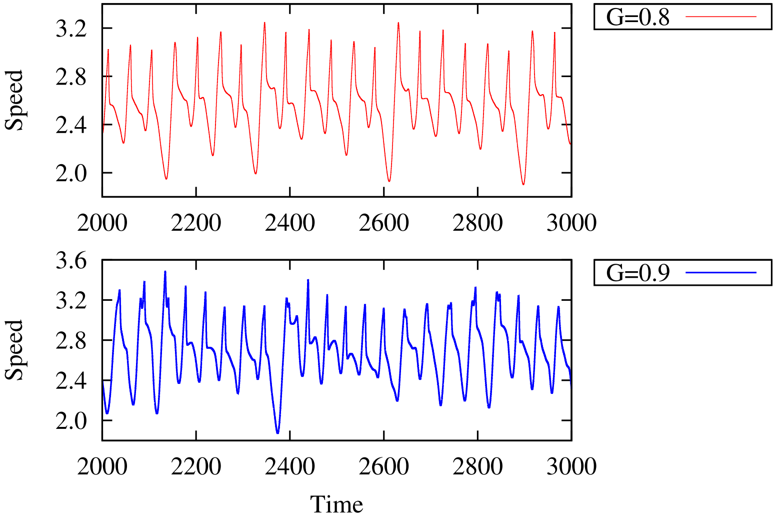


Period Doubling- Left/Right Symmetry Breaks



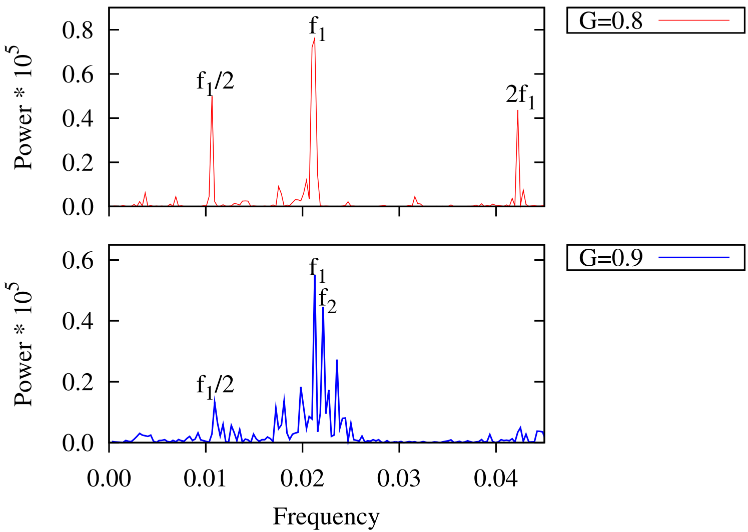
- **Base Period**: Up/down motion of the flame
- **Doubled Period**: Side-to-side motion

Torus bifurcation



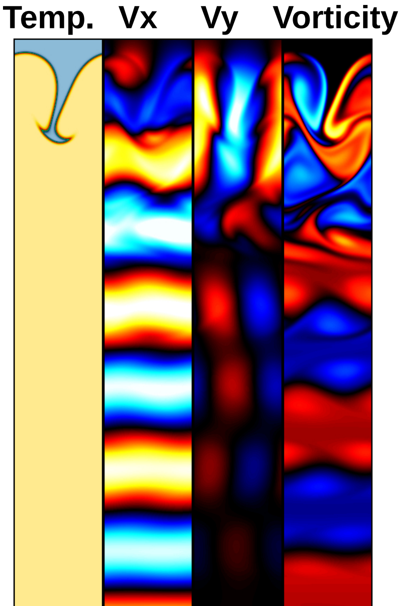
- An extra, incommensurate frequency develops.

Torus bifurcation: Power Spectra

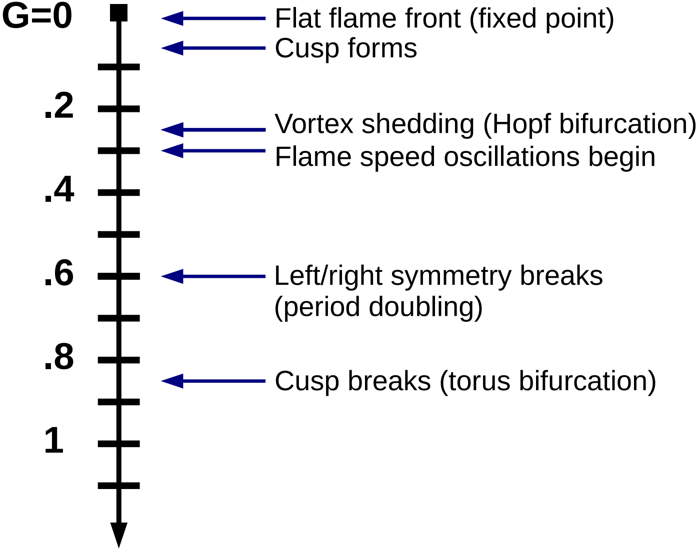


Torus Bifurcation: Cusp Breaking ($G = 0.9$)

- New Frequency introduced by **cusp breaking**
- The cusp is broken when the **Rayleigh-Taylor instability** begins to overwhelm burning.



Summary for $G \leq 1$



Conclusions for low-G flames

Dynamical systems theory gives simple, but powerful, models for understanding complex-looking flame behaviors.

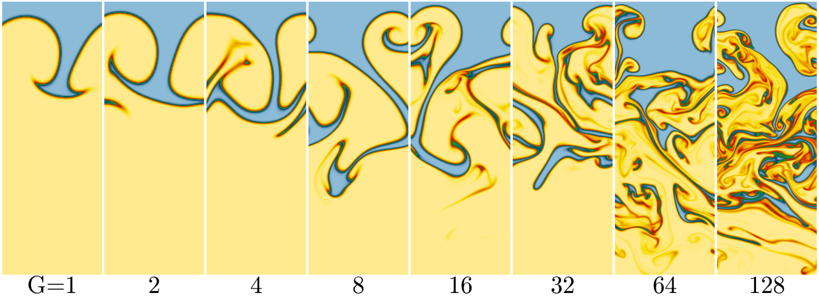
What causes changes in the surface area of the flame?

- At low G, the cusp creates vortex rolls which become unstable by a shear instability. These rolls affect the flame surface from behind.
- The flame front is disturbed by material behind it even at low values of G.

At what G does the Rayleigh-Taylor instability become important?

- R-T becomes directly important at $G = 0.9$

High G Flames ($G \geq 1$)

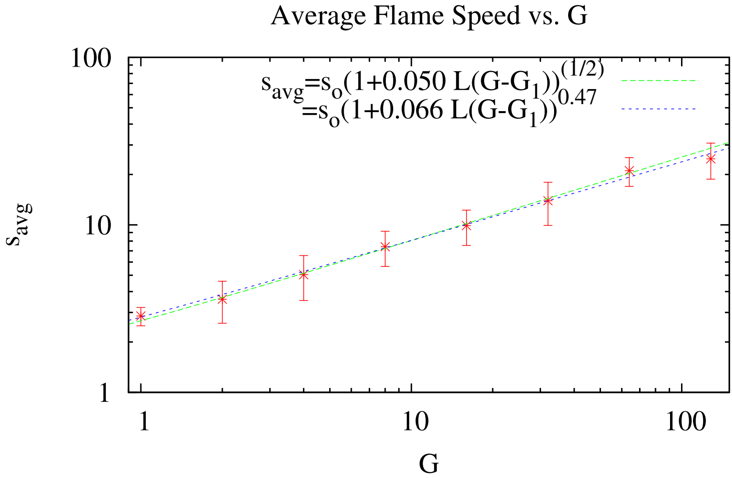


Two important questions:

1. What is the flame speed?
2. How well is turbulence able to wrinkle the flame front?

Flame Speed Scaling

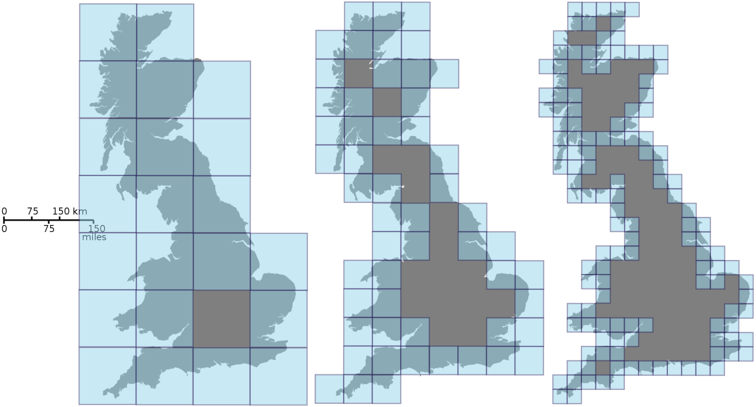
The average flame speed scales as $s \propto \sqrt{GL}$.



Subgrid models should be based on the Rayleigh-Taylor flame speed!

Box-Counting (Fractal) Dimension

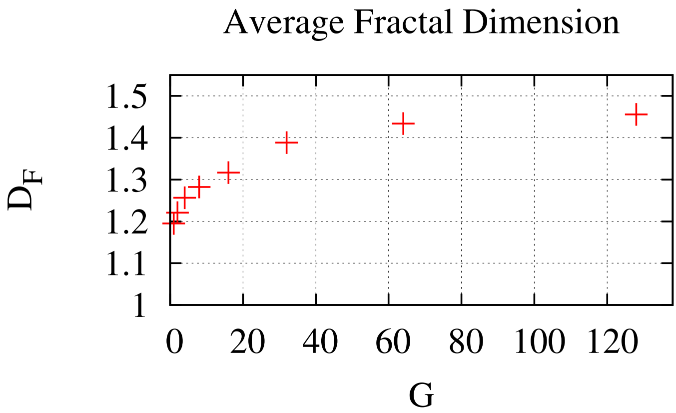
$$D_{frac} = -\lim_{\epsilon \to 0} \frac{log(N(\epsilon))}{log(\epsilon)}$$



(Mandelbrot 1967)

Image: Wikipedia

Measuring Flame Wrinkling- Fractal Dimension



- The flame front is wrinkled by turbulence, but the amount of wrinkling levels off.
- **Claim: At high values of G, large-scale stretching of the flame by the RT instability controls the flame dynamics.**

Fractal Dimension Model for the Flame Speed

The flame speed is proportional to the flame area:

$$\frac{s}{s_o} = \frac{L_f}{L}$$

The length of the flame is determined by:

- Large-scale Rayleigh-Taylor stretching
- Wrinkling at all scales by turbulence

What are the contributions from each of these effects?

A model for the flame area:

$$L_f = L_{RT}(G) * \left(\frac{L}{\eta(G)}\right)^{D_F - 1}$$

$L =$ the size of the largest eddies (the box size)
$\eta =$ the size of the smallest eddies (the Kolmogorov cutoff scale)

Some useful relations:

- For large values of G, $D_F \rightarrow 1.5$.
- For 2D turbulence, $\eta = L * Re^{-1/2}$.
- $Re = L(1 + 0.2L(G - G_1))^{0.56}$

Altogether this gives:

$$L_f \propto L_{RT}(G) * G^{0.14}$$

We already know that $s \propto \sqrt{GL}$ so:

- Rayleigh-Taylor stretching scales as: $G^{0.36}$
- Turbulent wrinkling scales as: $G^{0.14}$

Large-scale Rayleigh-Taylor stretching dominates the flame speed for high values of G.

Conclusions

- At low G, flames can be described by simple, low-dimensional bifurcation models.
- The initial disturbance of the flame surface is due to a shear instability behind the flame front.
- At large G, the Rayleigh-Taylor instability controls the burning rate by stretching the flame front.
- Subgrid models should be based on the Rayleigh-Taylor flame speed.
- The DDT transition is not possible for flames under these conditions.

Nek5000

DNS/LES computational fluid dynamics solver

Developers: Paul Fischer (chief architect), Aleks Obabko, James Lottes, Stefan Kerkemeier, Katie Heisey

Features:

- Spectral Element Code
- Incompressible or low-Mach number
- Very fast and efficient
- Low memory use
- Scales up to $100,000$ processors
- Works very well with solid boundaries

Website: http://nek5000.mcs.anl.gov

Flame Simulations

Parameters-WD

ρ_9	**G**	**L**	**GL**
10	$3.1 * 10^{-13}$	$7.9 * 10^{12}$	2.44
8	$6.4 * 10^{-13}$	$6.1 * 10^{12}$	3.9
6	$1.9 * 10^{-12}$	$4 * 10^{12}$	7.6
4	$9.5 * 10^{-12}$	$2 * 10^{12}$	19.5
2	$2.1 * 10^{-10}$	$5.4 * 10^{11}$	114
1	$6.5 * 10^{-9}$	$1.4 * 10^{11}$	910
.5	$2.1 * 10^{-7}$	$3.6 * 10^{10}$	7500
.2	$6.8 * 10^{-5}$	$4.9 * 10^{9}$	$3.3 * 10^{5}$
.1	$2 * 10^{-2}$	$1.2 * 10^{8}$	$2.5 * 10^{6}$
.05	$8.7 * 10^{-1}$	$4.3 * 10^{7}$	$3.8 * 10^{7}$
.01	$4.5 * 10^{2}$	$1.2 * 10^{7}$	$5.2 * 10^{9}$

Parameters-Simulations

G	**L**	**GL**
.25	128	32
.5	128	64
1	128	128
2	128	256
4	128	512
8	128	1024
16	128	2048
32	128	4096
64	128	8192
128	128	16384

The Equations

- Use the Boussinesq Approximation

Equations

Navier-Stokes Equation:

$$\frac{D\mathbf{u}}{Dt} = -\left(\frac{1}{\rho_o}\right)\nabla p + \frac{\rho\mathbf{g}}{\rho_o} + \nu\nabla^2\mathbf{u}$$

Heat Equation:

$$\frac{DT}{Dt} = \kappa\nabla^2 T + R(T)$$

Continuity Equation:

$$\nabla \cdot \mathbf{u} = 0$$

Bistable Reaction:

$$R(T) = 2T^2(1 - T)$$

Definitions

Laminar Flame Speed:

$$s_o = \sqrt{\alpha\kappa}$$

Flame Width:

$$\delta = \sqrt{\frac{\kappa}{\alpha}}$$

κ is the thermal diffusivity

$\frac{1}{\alpha}$ is the reaction time

Non-Dimensional Equations

Equations

Navier-Stokes Equation:

$$\frac{D\mathbf{u}}{Dt} = -\left(\frac{1}{\rho_o}\right)\nabla p + GT + Pr\nabla^2\mathbf{u}$$

Heat Equation:

$$\frac{DT}{Dt} = \nabla^2 T + 2T^2(1-T)$$

Continuity Equation:

$$\nabla \cdot \mathbf{u} = 0$$

Control Parameters

Non-Dimensional Gravity:

$$G = g\left(\frac{\triangle\rho}{\rho_o}\right)\frac{\delta}{s_o^2}$$

Prandtl Number:

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Non-Dimensional Box Size:

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