Complete WMAP Constraints on Inflationary Features

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Outline

- Inflation overview
- General method to constrain the inflationary potential from CMB observations allowing for features
  - Theoretical framework
  - Analysis of data
- Conclusions and future directions

Inflation *A.Guth, PRD (1981)*

- A period of accelerated expansion: $\ddot{a} > 0$
- Explains why the universe is approximately homogeneous and spatially flat.

Sourced by a **negative pressure:**

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) > 0 \Leftrightarrow \boxed{p < -\frac{\rho}{3}}$$

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What causes inflation?

The Dynamics of Inflation

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2$$

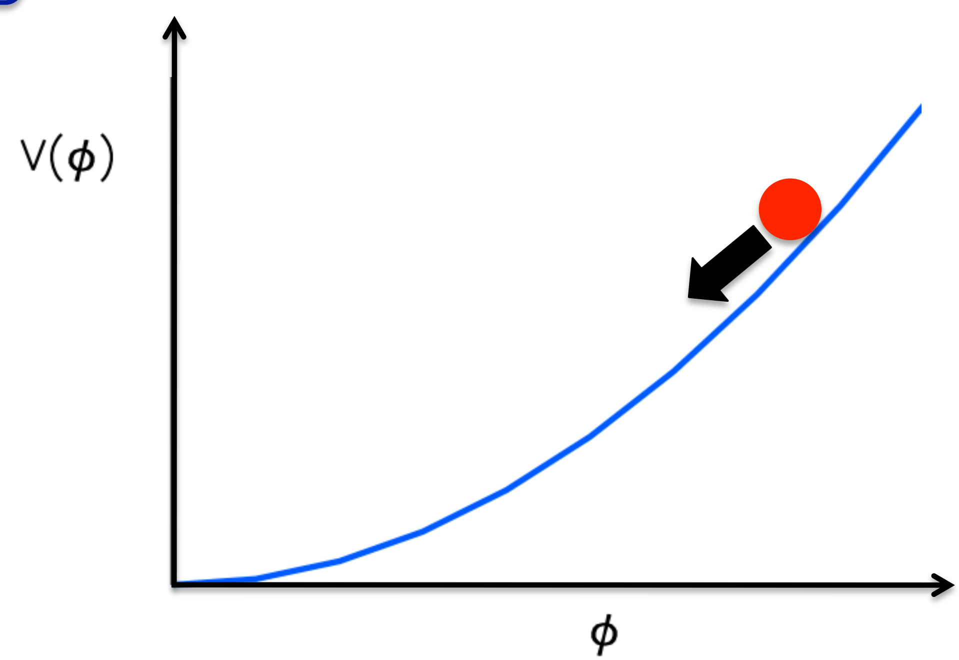
expansion rate

$$= \frac{8\pi G}{3}\left[\frac{1}{2}\dot{\phi}^2 + V\left(\phi\right)\right]$$

energy density

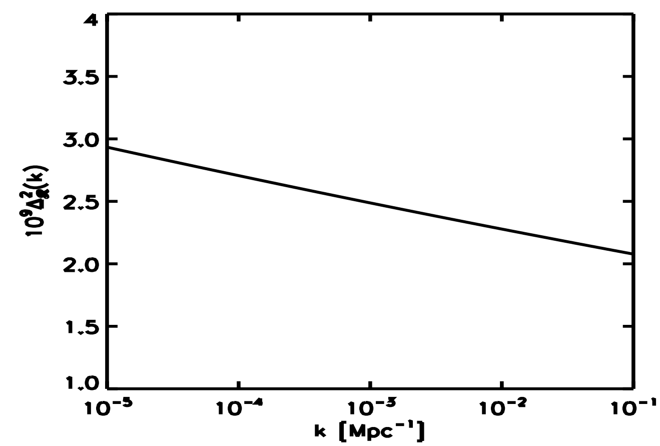
$$\ddot{\phi} + 3H\dot{\phi} + V' = 0$$

friction

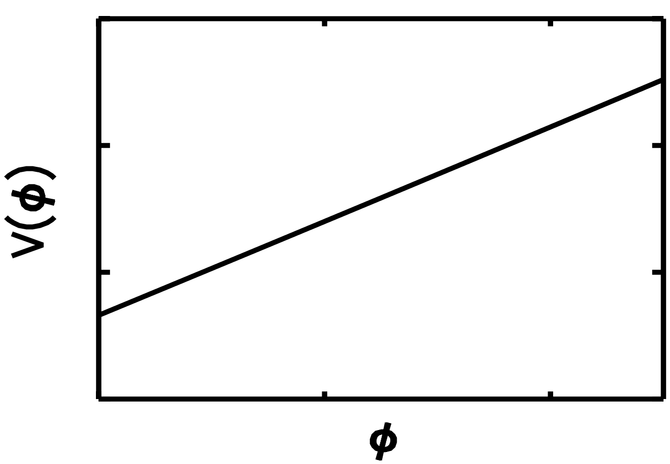


Connecting Theory with Observations

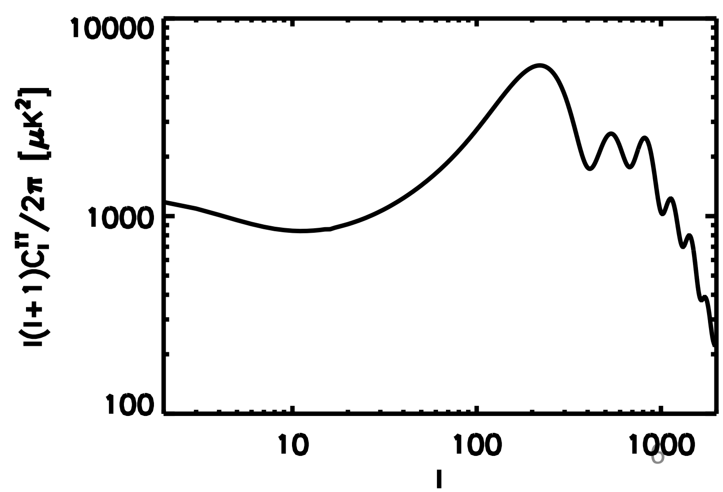
POWER SPECTRUM



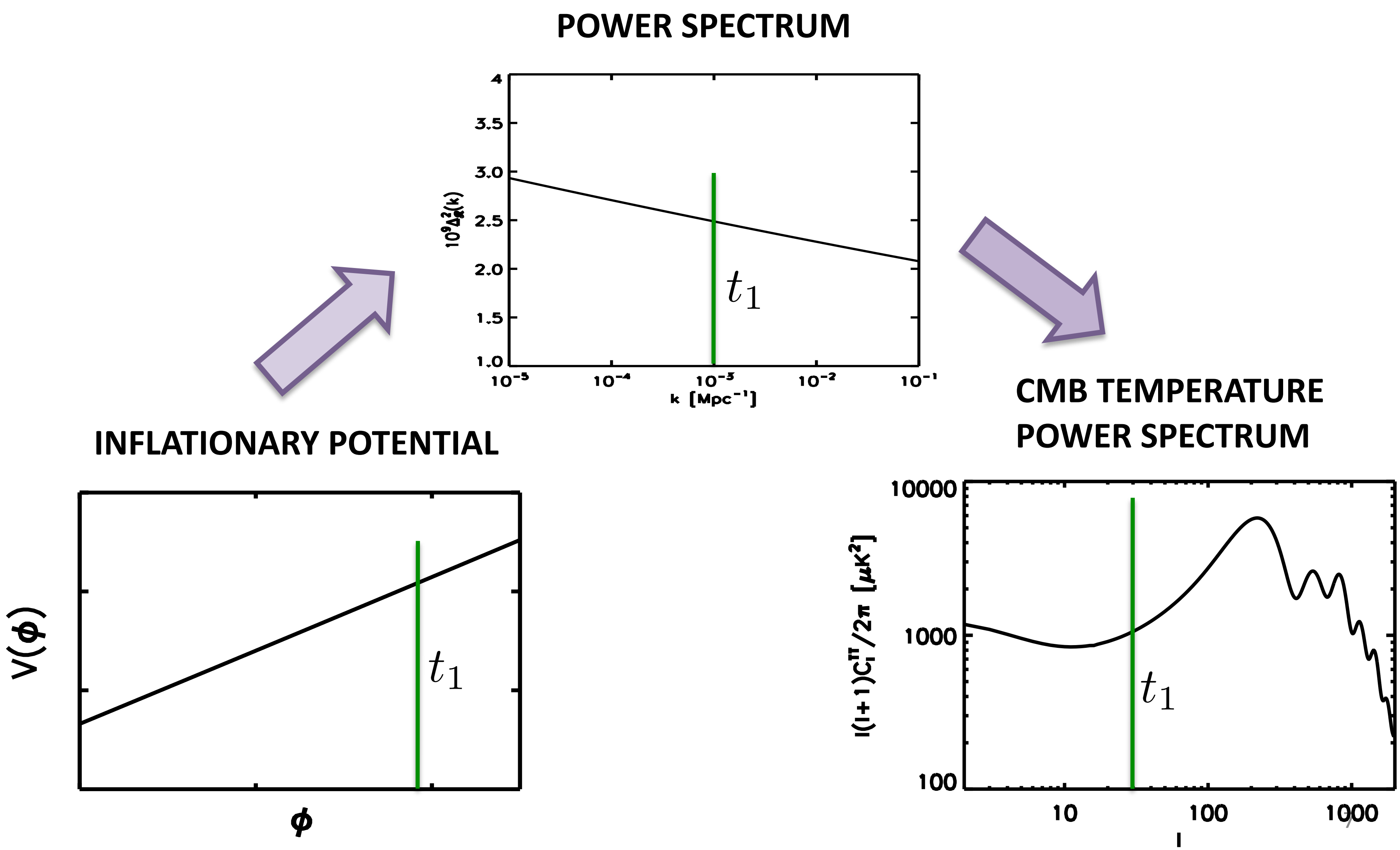
INFLATIONARY POTENTIAL



CMB TEMPERATURE POWER SPECTRUM

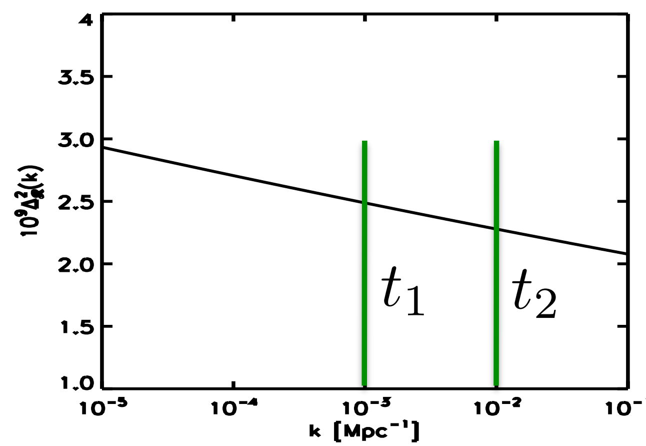


Connecting Theory with Observations

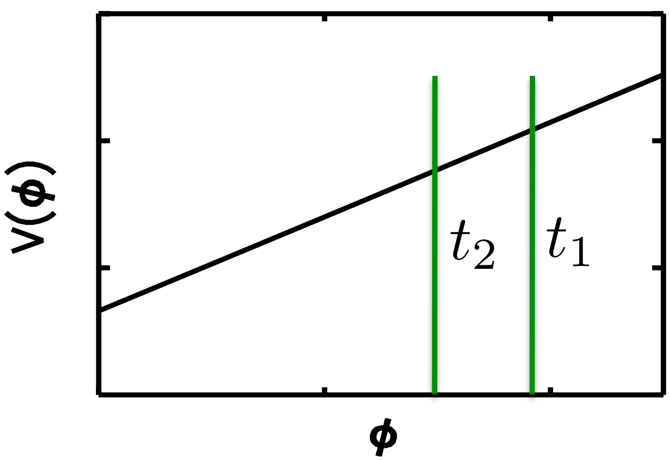


Connecting Theory with Observations

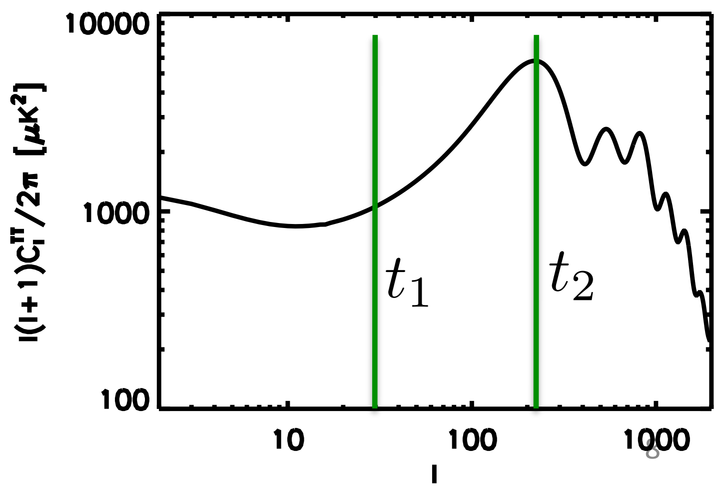
POWER SPECTRUM



INFLATIONARY POTENTIAL



CMB TEMPERATURE POWER SPECTRUM



Connecting Theory with Observations

Goal: to shed light on the physics of inflation by using CMB observations

Standard Slow Roll

Technique for computing the initial curvature power spectrum $\Delta^2_{\mathcal{R}}$ for inflationary models where the scalar field potential is sufficiently flat and slowly varying.

Linked to the **shape of the potential**

$$\epsilon_H \equiv \frac{1}{2}\left(\frac{\dot{\phi}}{H}\right)^2$$

$$\eta_H \equiv -\left(\frac{\ddot{\phi}}{H\dot{\phi}}\right)$$

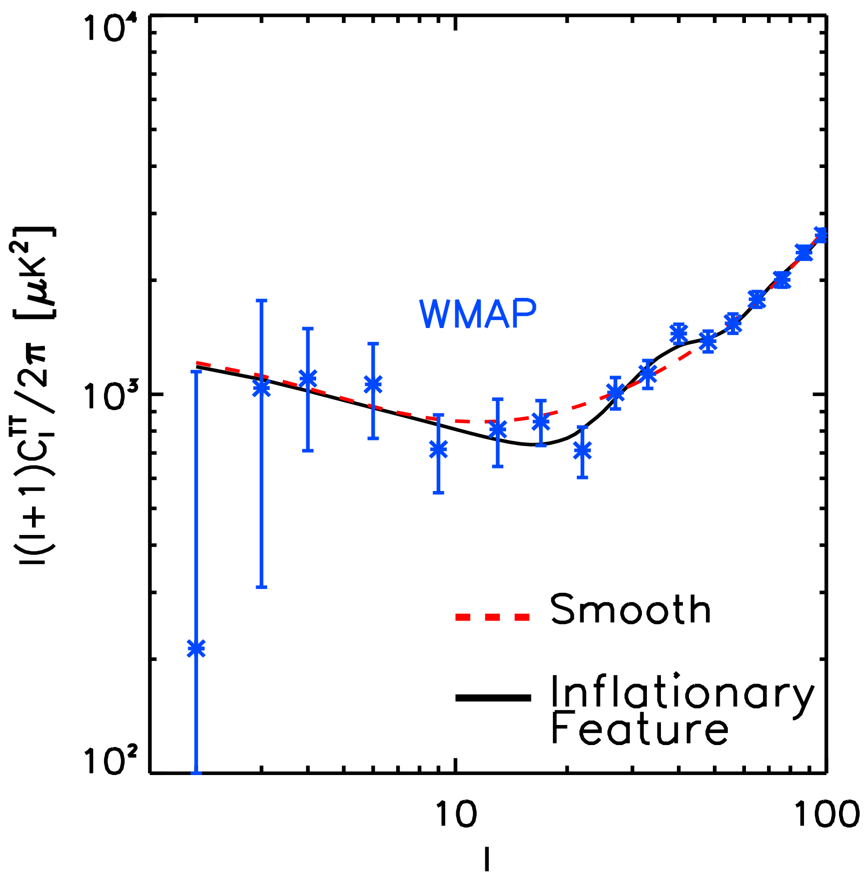
$$\delta_2 \equiv \frac{\dddot{\phi}}{H^2\dot{\phi}}$$

Slow-roll parameters

<< 1 and slowly varying

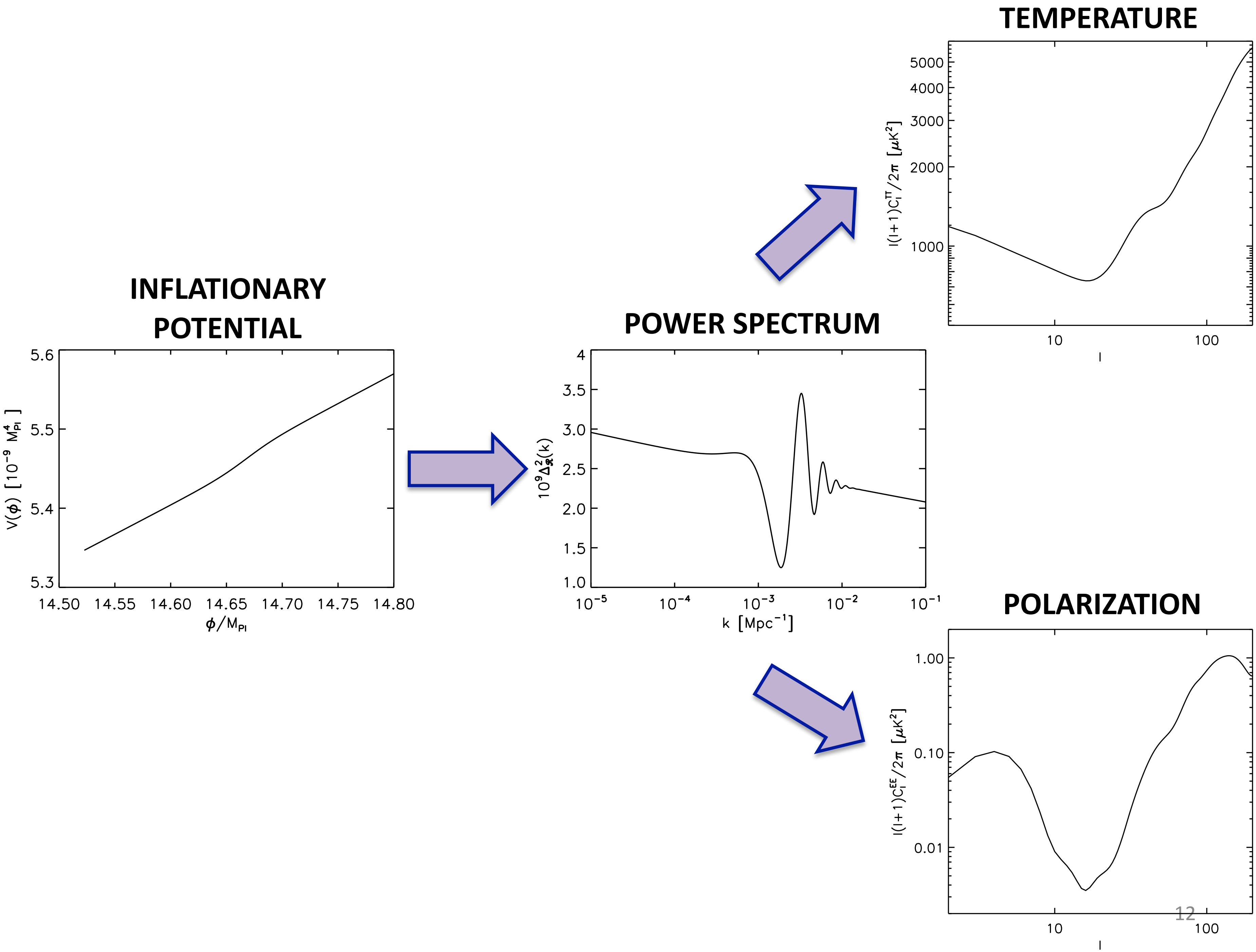
Slow roll approximation: $\Delta^2_{\mathcal{R}} \approx \left[(1-(2C+1)\epsilon_H - C\eta_H)\,\frac{H^2}{2\pi|\dot{\phi}|}\right]^2_{k\eta\approx 1}$

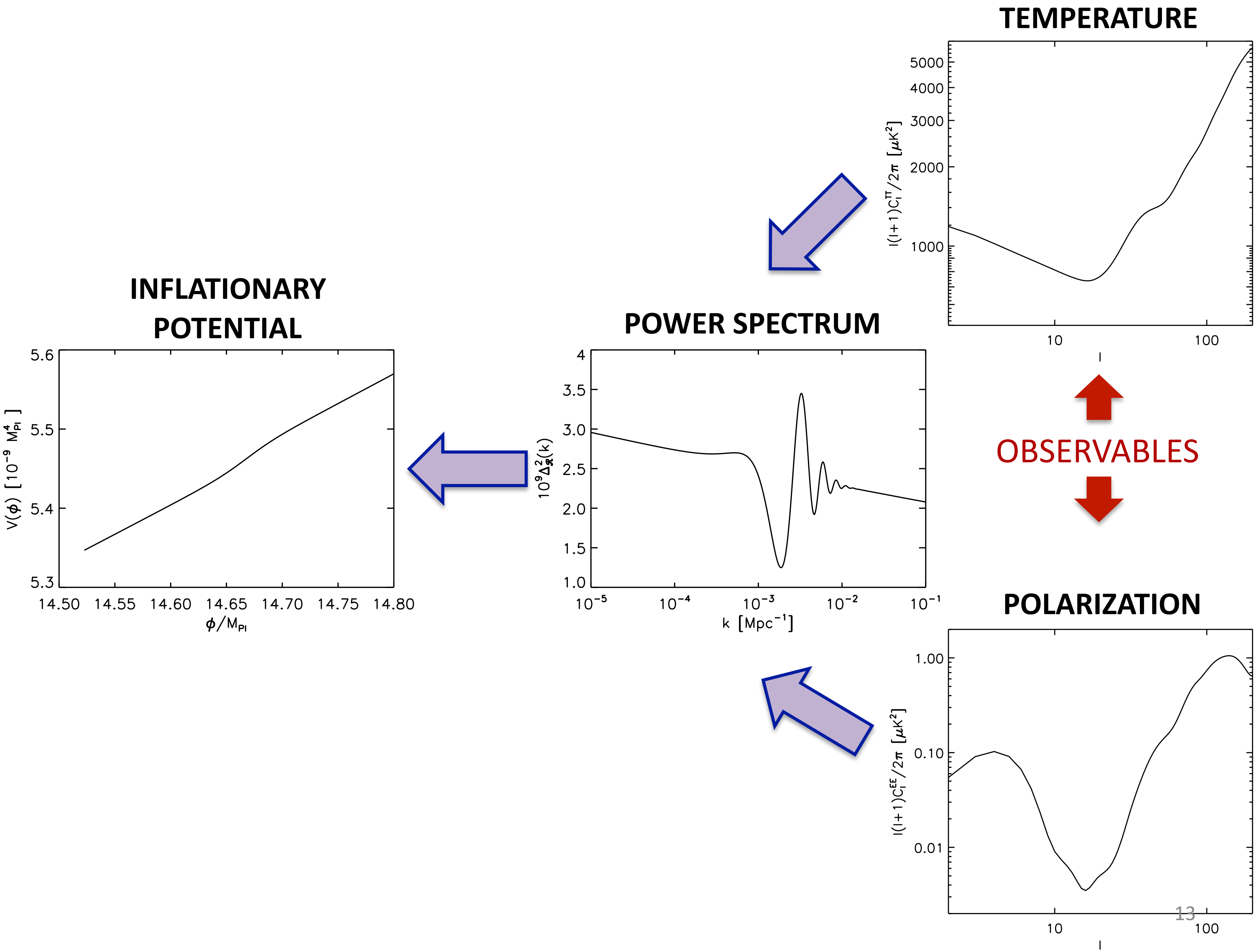
Inflationary Features



- Glitches in the temperature power spectrum of the CMB have led to interest in exploring models with features in the inflaton potential.

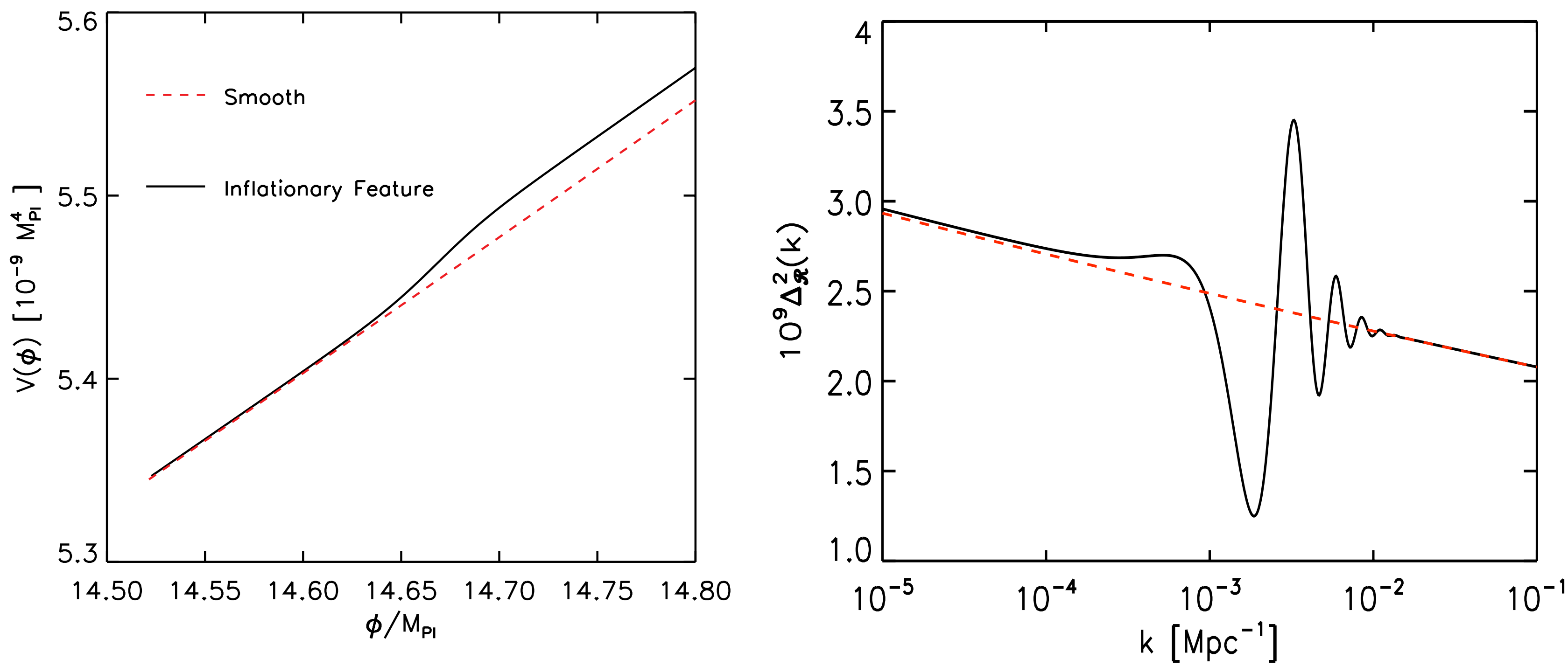
✧ ***L.Covi, J.Hamann, A.Melchiorri, A.Slozar and I.Sorbera, (2006)***
✧ ***M.Mortonson, C.Dvorkin, H.V.Peiris and W.Hu, PRD (2009)***


INFLATIONARY POTENTIAL
V(ϕ) [10⁻⁹ M_Pl⁴]
ϕ/M_Pl
POWER SPECTRUM
10⁹Δ²_R(k)
k [Mpc⁻¹]
TEMPERATURE
l(l+1)C_l^TT/2π [μK²]
l
POLARIZATION
l(l+1)C_l^EE/2π [μK²]
l


TEMPERATURE
INFLATIONARY POTENTIAL
POWER SPECTRUM
OBSERVABLES
POLARIZATION
V(ϕ) [10⁻⁹ M_Pl⁴]
ϕ/M_Pl
10⁹Δ²_R(k)
k [Mpc⁻¹]
l(l+1)C_l^TT/2π [μK²]
l(l+1)C_l^EE/2π [μK²]
l

Inflationary Features

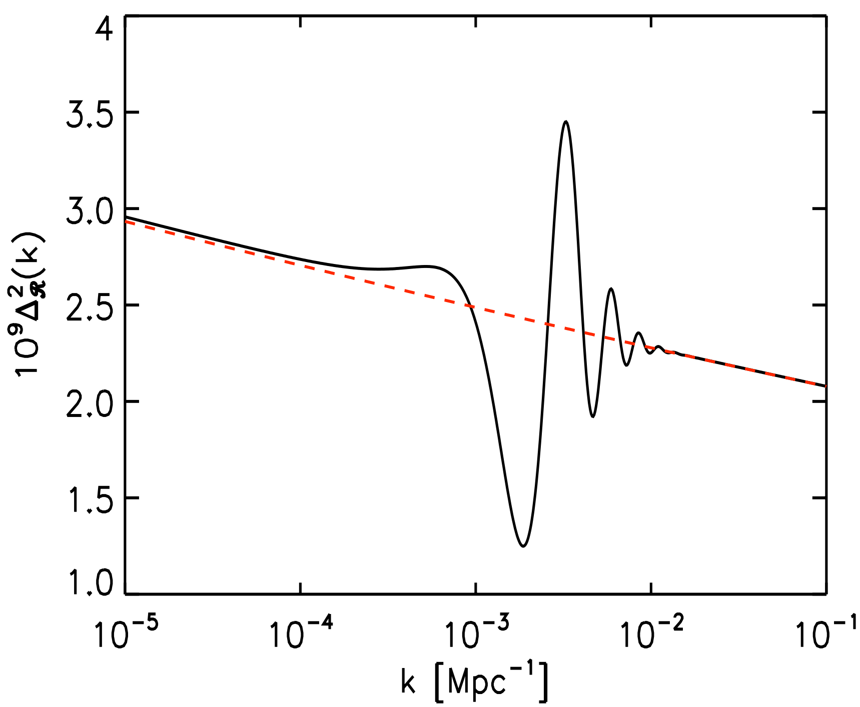
The rolling of the inflaton across the feature produces ringing in the power spectrum.



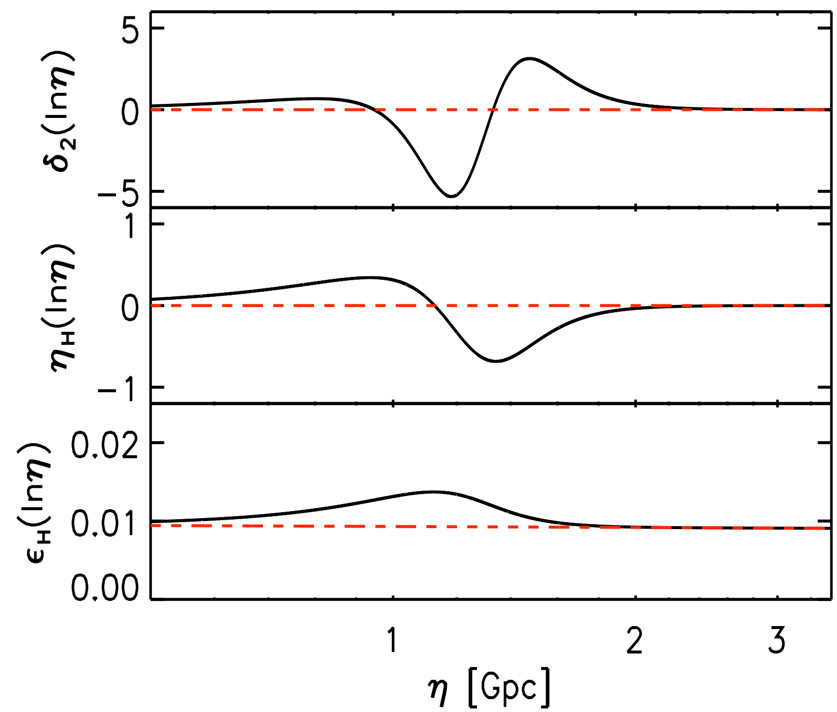
M.Mortonson, C.Dvorkin, H.V.Peiris and W.Hu, PRD (2009)

Breaking Slow Roll

- These models require order unity variations in the curvature power spectrum: slow-roll parameters are not necessarily small or slowly varying.



Standard slow roll:
not a valid approximation!



C.Dvorkin, W.Hu, PRD (2009)

Generalized Slow Roll

E. Stewart, PRD (2002)

- Field equation: $$\frac{d^2y}{dx^2} + \left(1 - \frac{2}{x^2}\right) y = \frac{g(\ln x)}{x^2} y$$

  $(y = \sqrt{2k}u_k;\ x = k\eta)$

  Source function

- de-Sitter solution: $$\frac{d^2y_0}{dx^2} + \left(1 - \frac{2}{x^2}\right) y_0 = 0$$

- GSR approximation: $$\frac{d^2y}{dx^2} + \left(1 - \frac{2}{x^2}\right) y = \frac{g(\ln x)}{x^2} y_0$$

Solution can be constructed with a Green function approach.

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  Solution can be constructed with a Green function approach.

BUT…

- Nodes in the power spectrum.
- Curvature is not constant for modes outside the horizon.

Our GSR solution for large features

- The curvature power spectrum depends on a single source function:

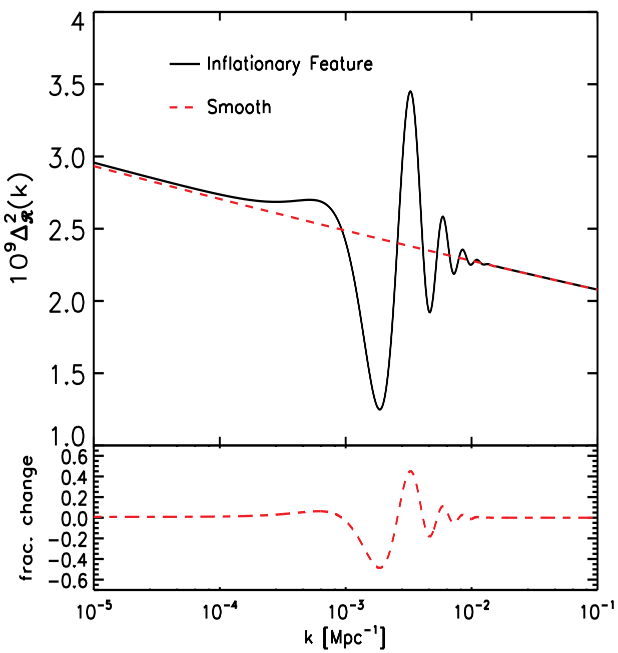
$$\ln \Delta^2_{\mathcal{R}}(k) = G(\ln \eta_{\rm min}) + \int_{\eta_{\rm min}}^{\eta_{\rm max}} \frac{d\eta}{\eta} W(k\eta) G'(\ln \eta) + \ln\left[1 + \frac{1}{2}\left(\int_{\eta_{\rm min}}^{\eta_{\rm max}} \frac{d\eta}{\eta} X(k\eta) G'(\ln \eta)\right)^2\right]$$

C.Dvorkin, W.Hu, PRD (2009)

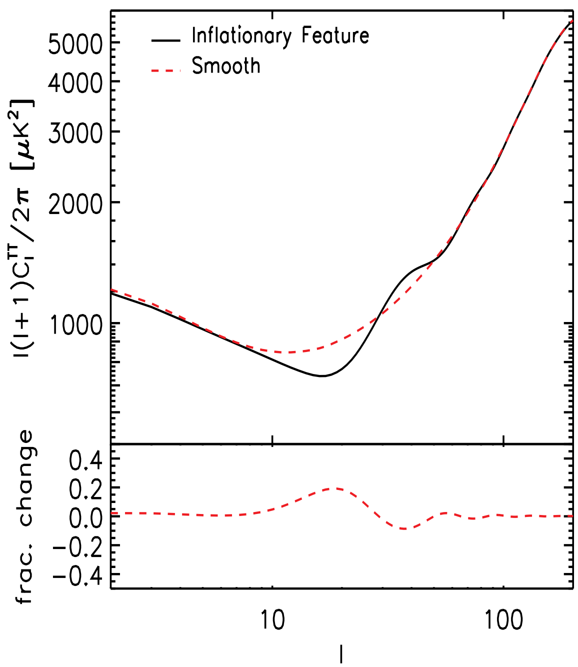
✓ Constant curvature for modes outside the horizon.
✓ We recover the slow-roll result for a constant source.
✓ Well controlled for time varying and order unity slow-roll parameters: percent level errors.
✓ Simple to relate to the inflaton potential: $G' \approx 3\left(\frac{V_{,\phi}}{V}\right)^2 - 2\left(\frac{V_{,\phi\phi}}{V}\right)$

Second order Generalized Slow Roll: Well controlled

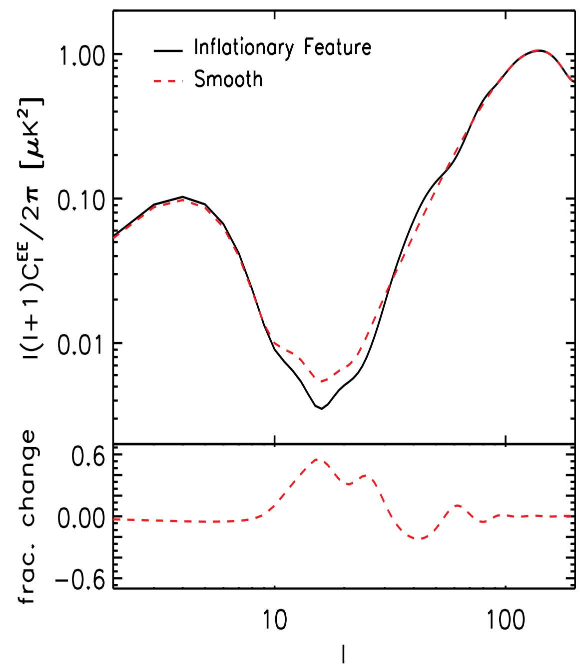
POWER SPECTRUM



TEMPERATURE



POLARIZATION



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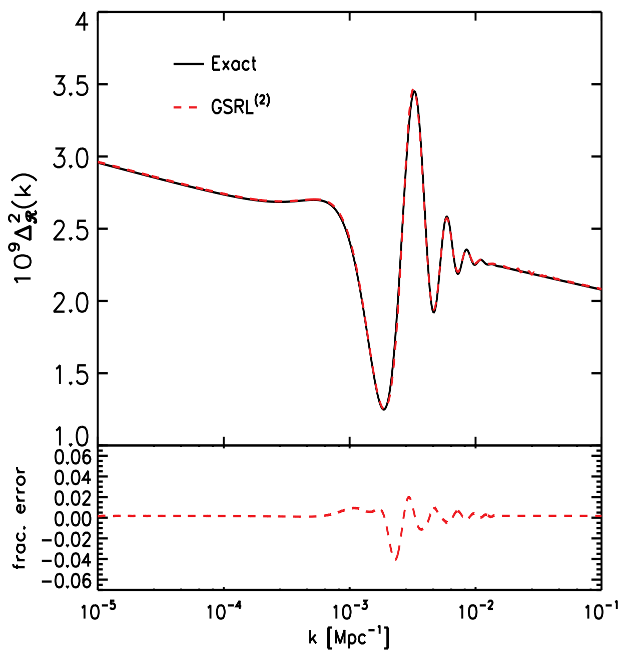
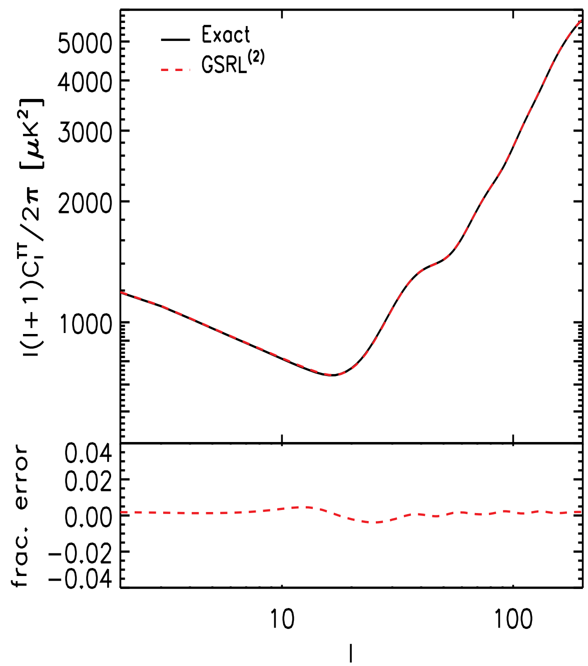
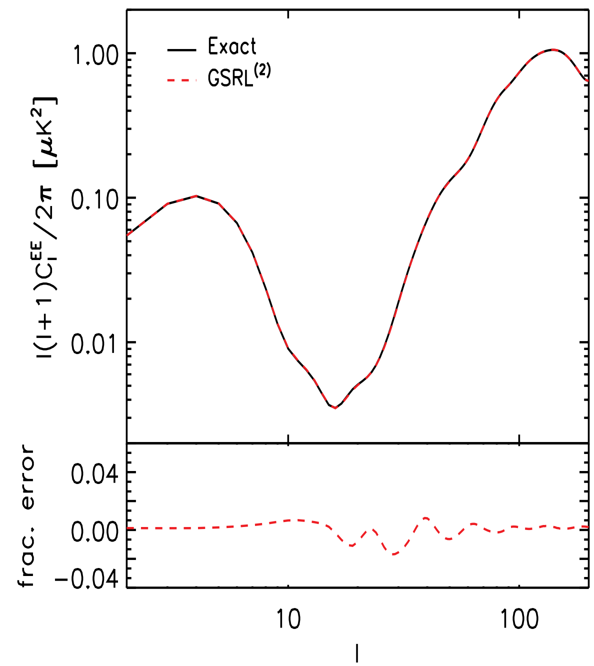


TEMPERATURE



POLARIZATION

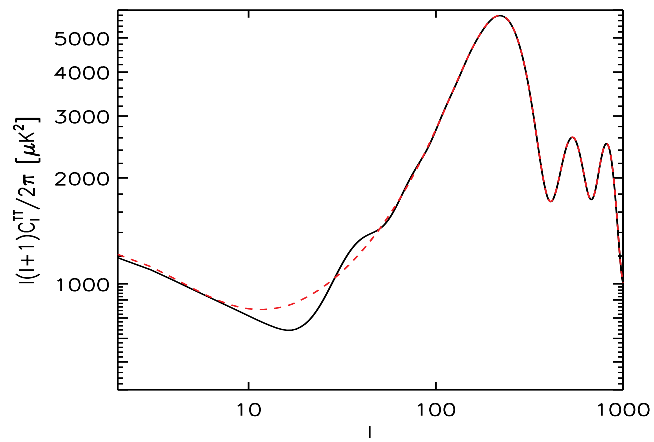


C.Dvorkin, W.Hu, PRD (2009)

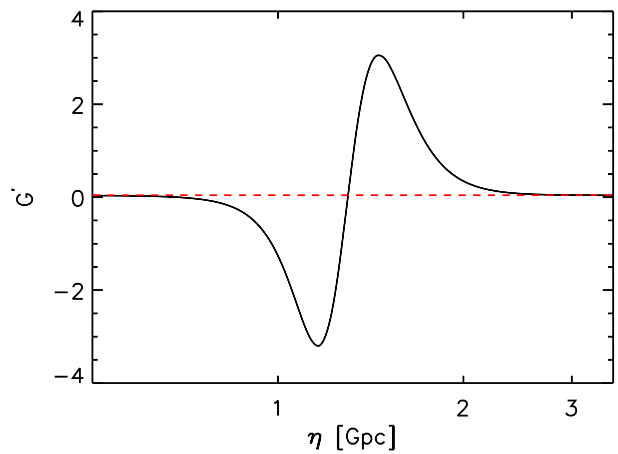
Accurate at <1% level for order unity features!

We can map observational constraints from the CMB onto constraints on the source…

- Power spectrum

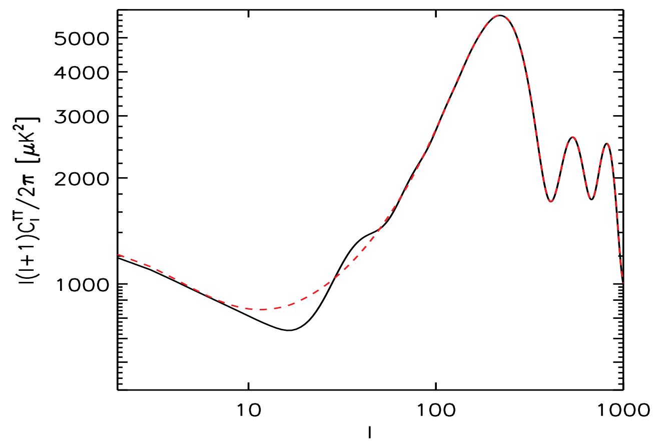


- Source

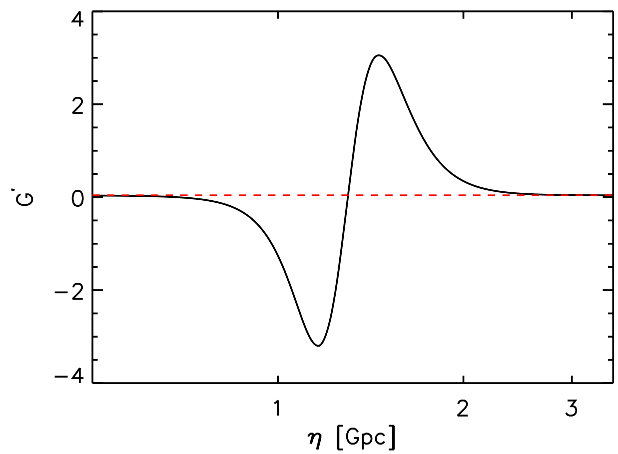


...and use these empirical constraints to test any model of single-field inflation.

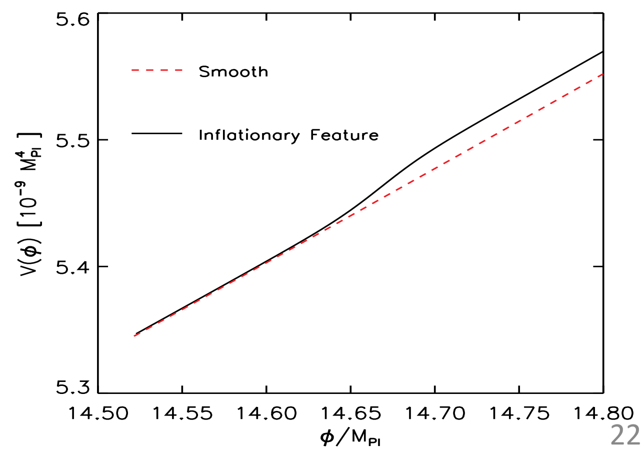
- Power spectrum



- Source



- Inflationary Model



Outline

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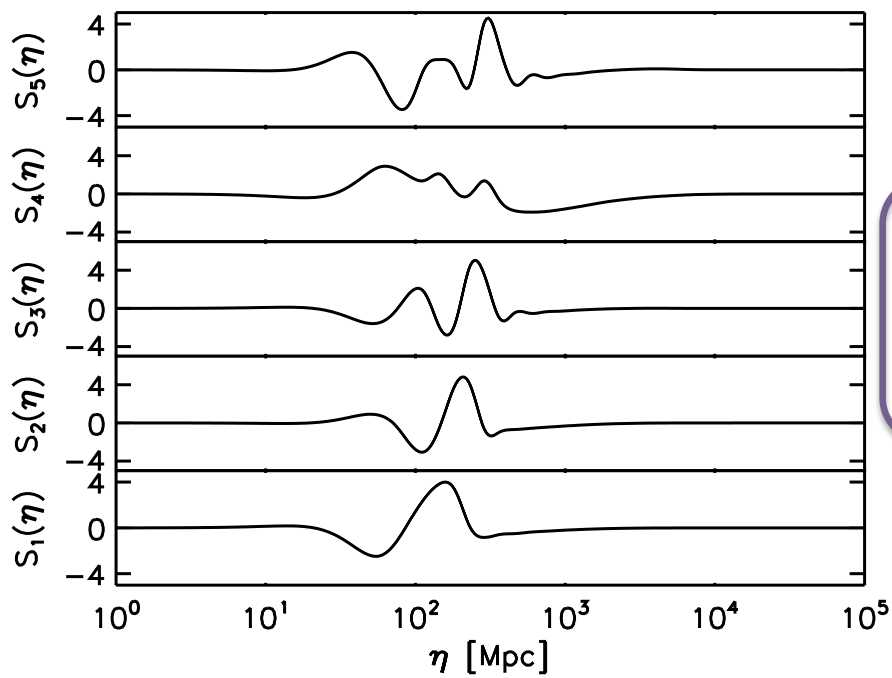
Model-independent constraints

Principal components *(of covariance matrix of perturbations in the source)*: basis for a complete representation of observable properties of the source function.

$$G' = 1 - n_s + \sum_{a=1}^{N} m_a S_a(\ln \eta)$$

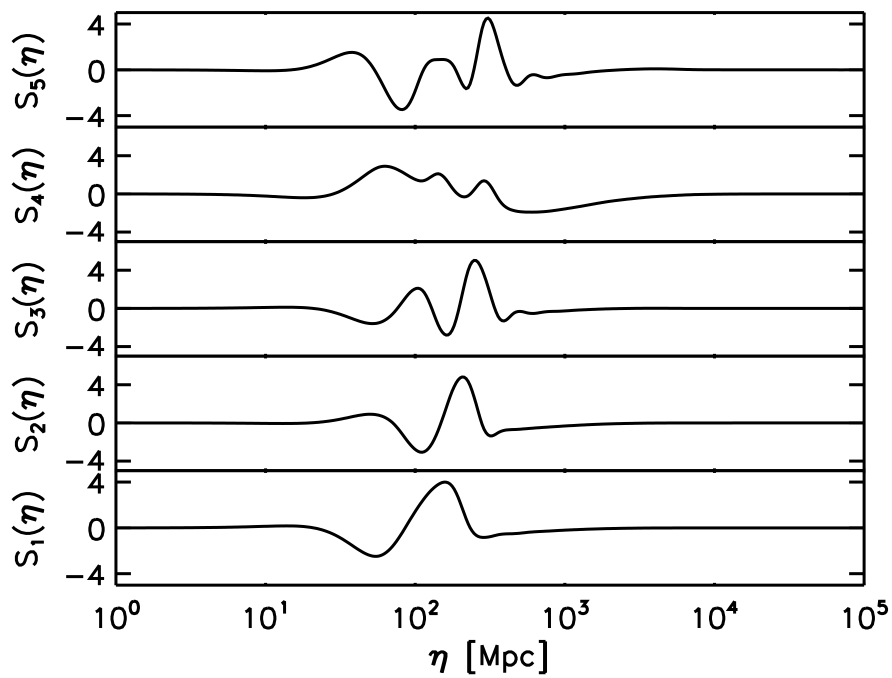


Defined a priori from covariance matrix: avoids *a posteriori* bias when looking at the data.

Model-independent constraints

Principal components *(of covariance matrix of perturbations in the source)*: basis for a complete representation of observable properties of the source function.

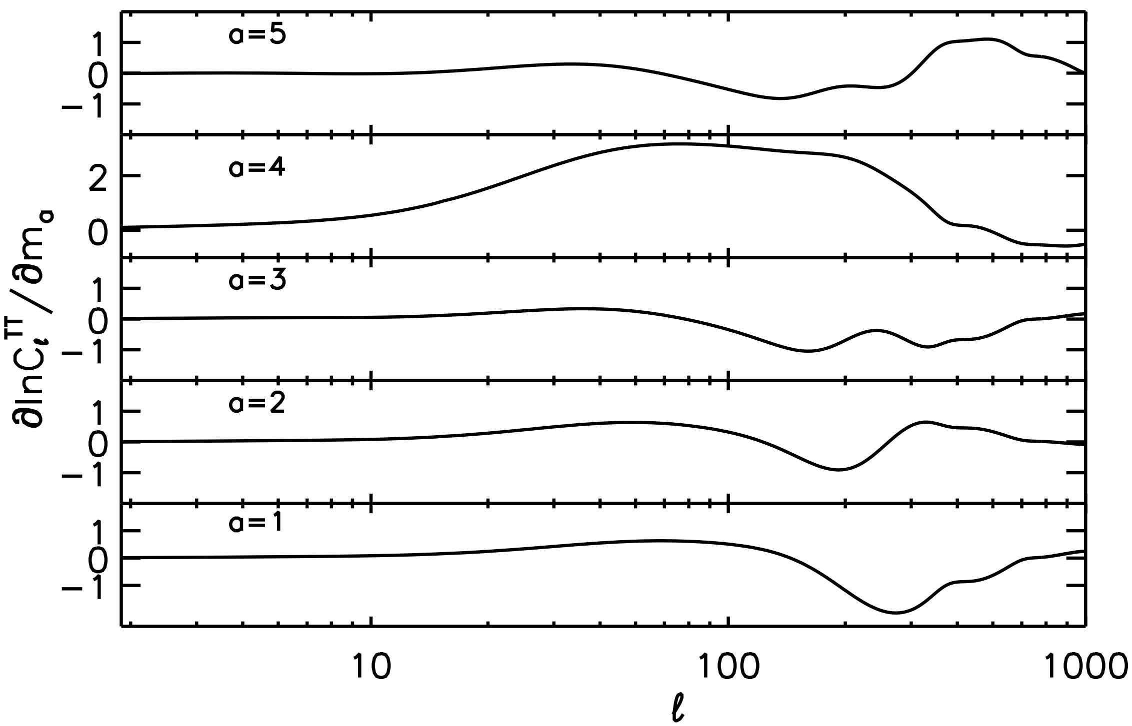
$$G' = 1 - n_s + \sum_{a=1}^{N} m_a S_a(\ln \eta)$$



- **Ranked in order of observability.**
- **Keep 5 best measured modes.**

Lower order PC's in WMAP

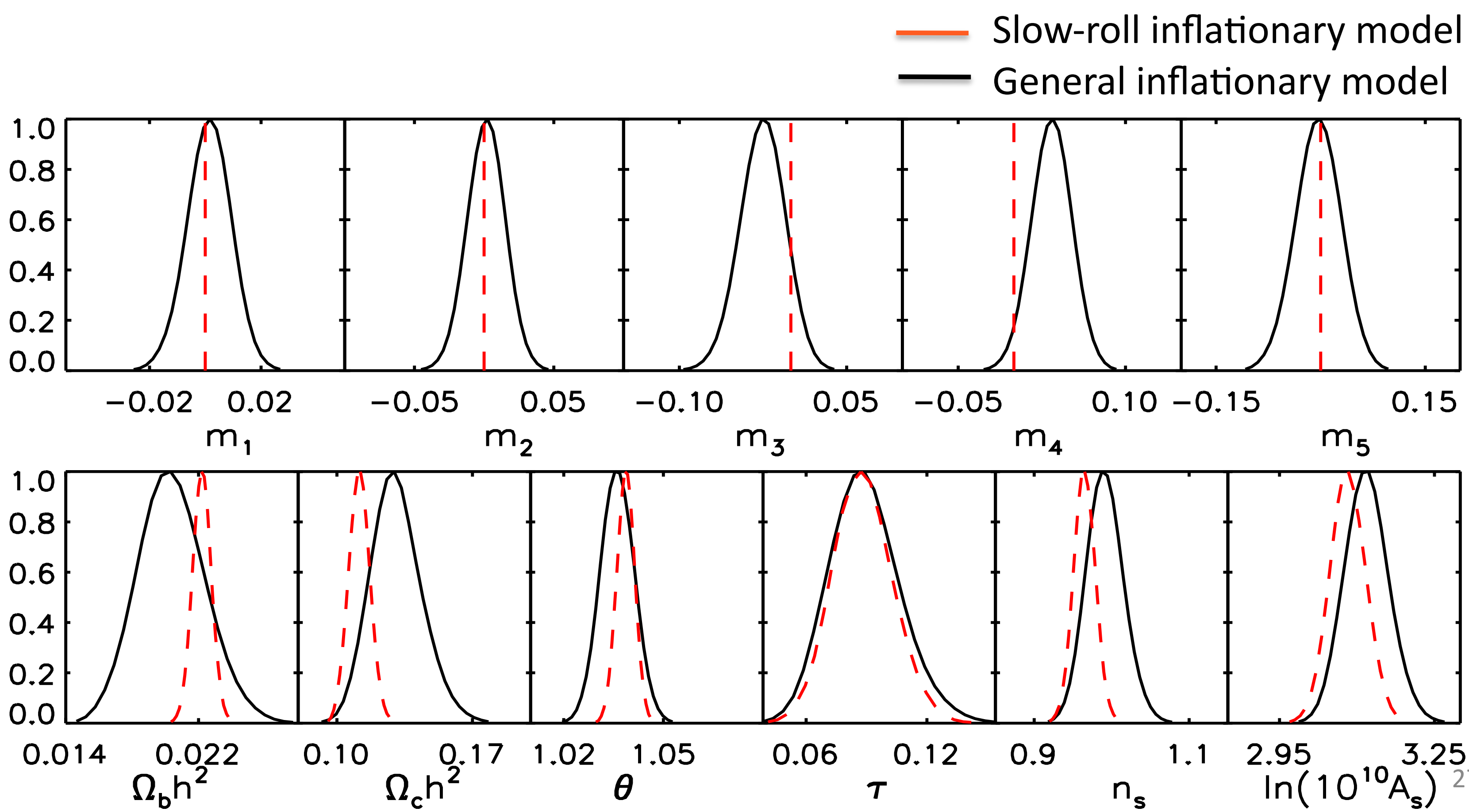
- Have their weight in the region best measured by the data (angular scales around the first acoustic peak, $\ell \approx 200$).



C.Dvorkin, W.Hu, PRD (2010)

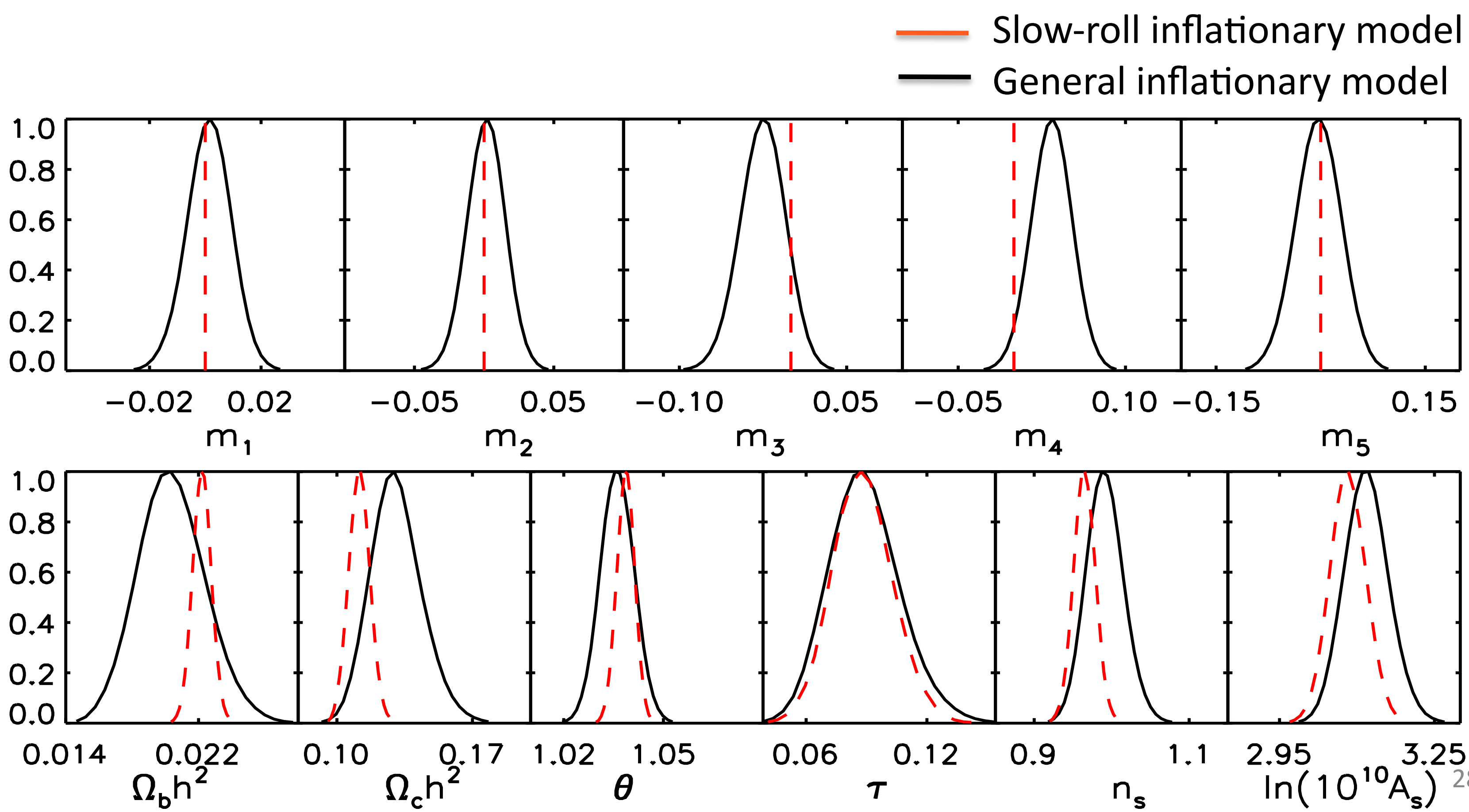
WMAP7 constraints on the first 5 PCs

- Non-zero values represent deviations from slow-roll and power-law spectrum.
- 1 out of 5 shows a 95% CL preference for a non-zero value, but only with a high cold dark matter density (which is disfavored by current SN and H0 data).

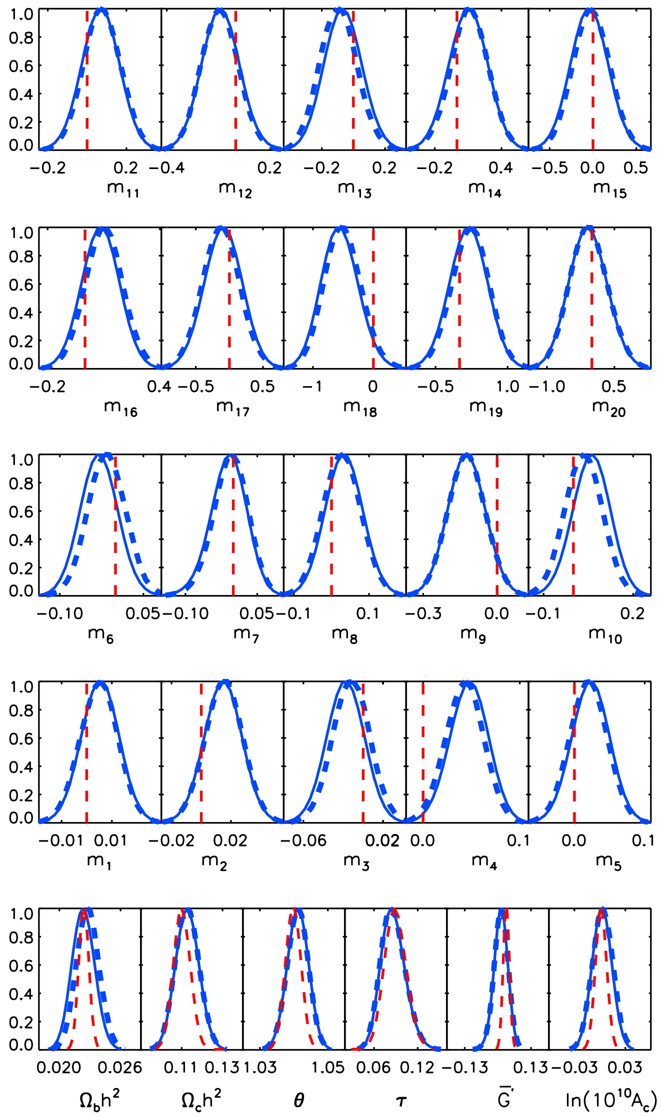


WMAP7 constraints on the first 5 PCs

- Consistency with a smooth inflationary potential: $\Delta\chi^2 \approx 5$ (with 5 additional parameters); robust to inclusion of tensor modes, spatial curvature and SZ emission.



Complete basis for Inflationary Features



- The fourth component is again the most discrepant mode.
- 3 components out of 20 exceed the 90% CL significance for nonzero value.

C.Dvorkin, W.Hu, PRD (2011)

Main bottleneck in the likelihood code:

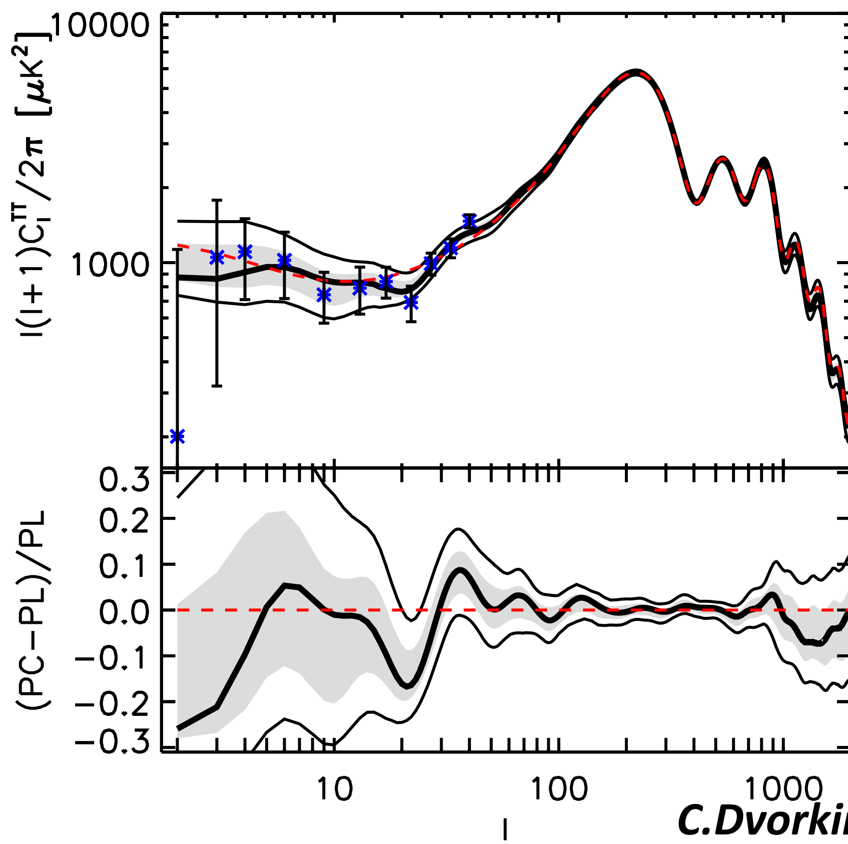
- OMP parallelized **WMAP likelihood code** and improved its speed by **~ 5*Ncore**

Publicly available:
http://background.uchicago.edu/wmap_fast/

WMAP7 + BICEP + QUAD data;
SN + H0 + BBN constraints.

Model-independent test of Slow Roll

- The ML model model only improves by $2\Delta \ln L = 17$ for the 20 additional parameters added.
- The marginal improvement is associated with features at $\ell < 60$.

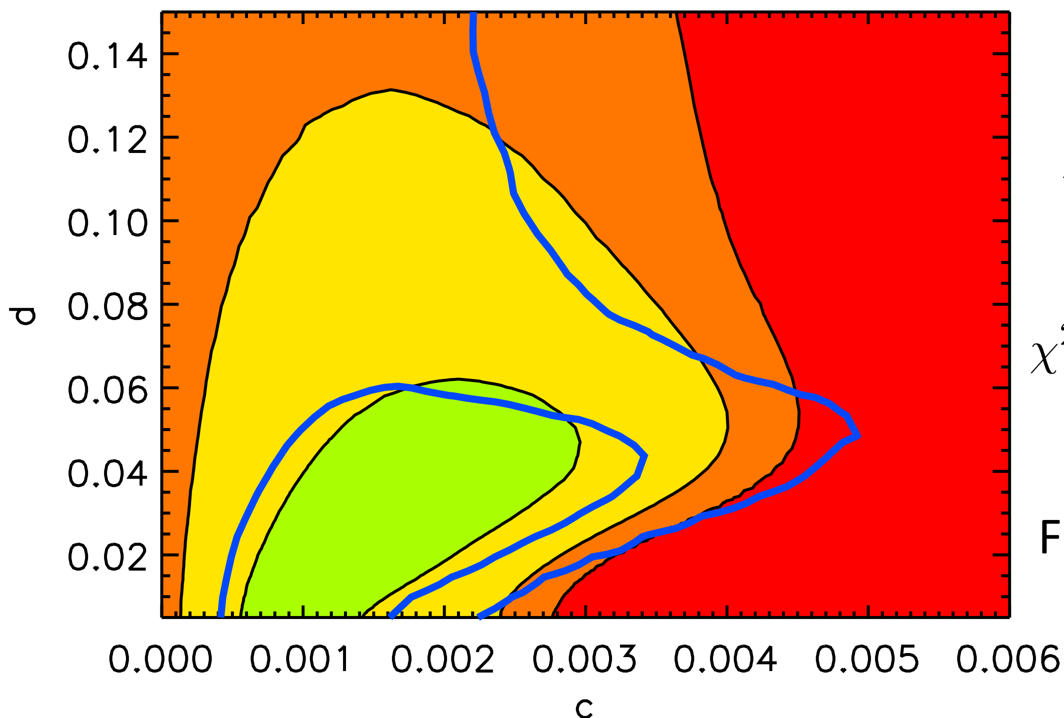


C.Dvorkin, W.Hu, PRD (2011)

Testing models of inflation

In practice, one can project any inflationary model onto the PC basis, and assess its significance using the means and covariance of our analysis:

$$\chi^2 = \sum_{a,b=1}^{20} \left[(m_a - \bar{m}_a)\mathbf{C}_{ab}^{-1}(m_b - \bar{m}_b)\right]$$



Example: step-potential

$$V(\phi) = \frac{1}{2}m^2\phi^2 \left[1 + c\tanh\left(\frac{\phi - \phi_s}{d}\right)\right]$$

χ^2 approx: c=0.0015, d=0.026

$$\Delta\chi^2 = -10.2$$

Full posteriors: c=0.0016, d=0.025

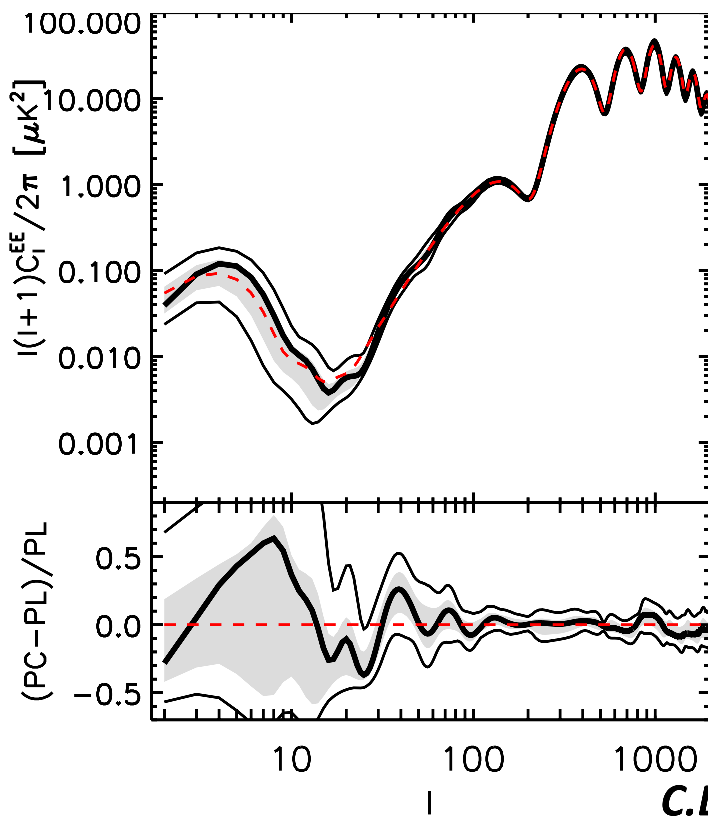
$$-2\Delta \ln L = -9.1$$

C.Dvorkin, W.Hu, PRD (2011)

The Predictive Power of Polarization

- Measurements at $\ell = 20 - 40$ (at the 40% level) will test the feature hypothesis at 2.5-3σ with Planck and 5-8σ with CMBPol. Caveat: confusion with reionization features.

M.Mortonson, C.Dvorkin, H.V.Peiris, W.Hu, PRD (2009)

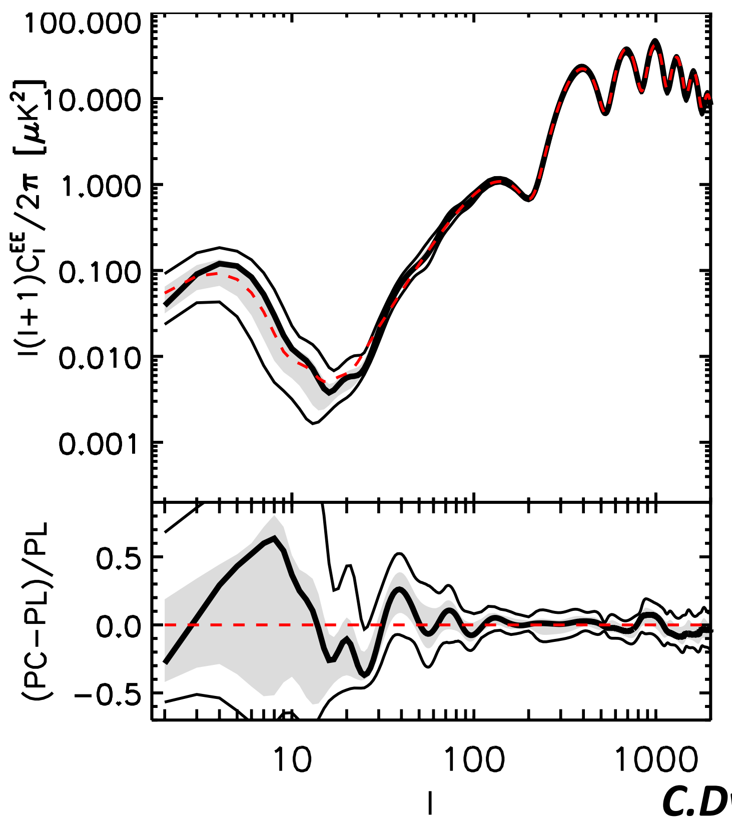


WMAP7 + BICEP + QUAD data; SN + H0 + BBN constraints.

C.Dvorkin, W.Hu, PRD (2011)

Model-independent test of single-field inflation

- Measurements lying outside these bounds could potentially rule-out single field inflation.



WMAP7 + BICEP + QUAD data; SN + H0 + BBN constraints.

C.Dvorkin, W.Hu, PRD (2011)

Conclusions and future directions

- Introduced a general formalism to constrain the inflationary potential from the data allowing for large amplitude and rapidly varying deviations from slow roll.

- Constraints around the first acoustic peak are consistent with a smooth inflationary potential. A complete analysis of inflationary features shows no significant deviations from slow roll.

- Matching features in the polarization power spectrum would test their inflationary origin.

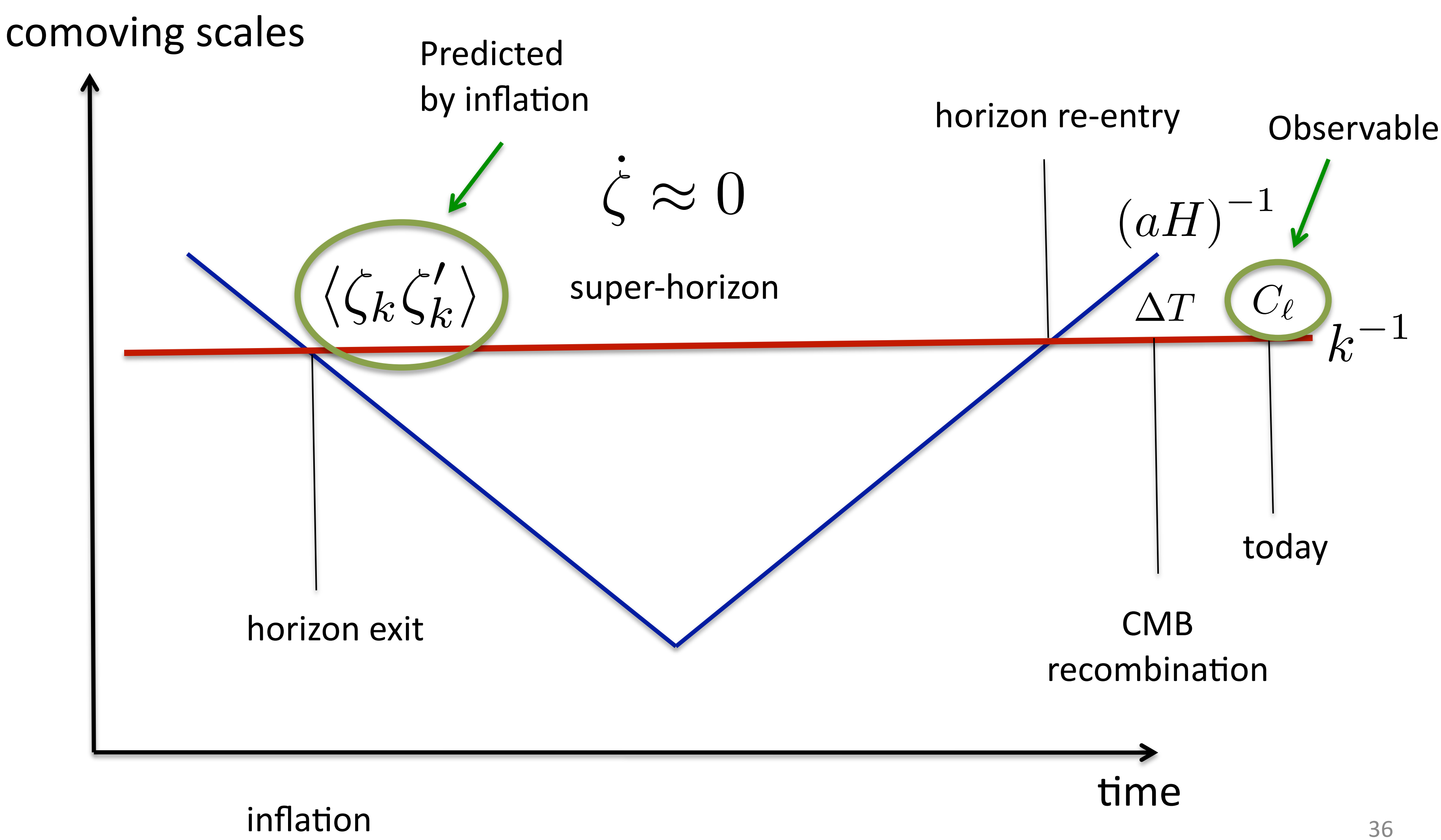
- Model-independent test of single-field inflation.

- This analysis can be used to constrain parameters of specific models inflation without requiring a separate likelihood analysis for each choice.

Work in progress:

- Construct an analogous formalism for calculating the bispectrum from the shape of the $V(\phi)$ potential.

Extra slides



comoving scales
Predicted
by inflation
horizon re-entry
Observable
$\dot{\zeta} \approx 0$
$(aH)^{-1}$
$\langle \zeta_k \zeta'_k \rangle$
super-horizon
ΔT
C_ℓ
k^{-1}
today
horizon exit
CMB
recombination
time
inflation

Generalized Slow Roll

E.Stewart, PRD (2002)

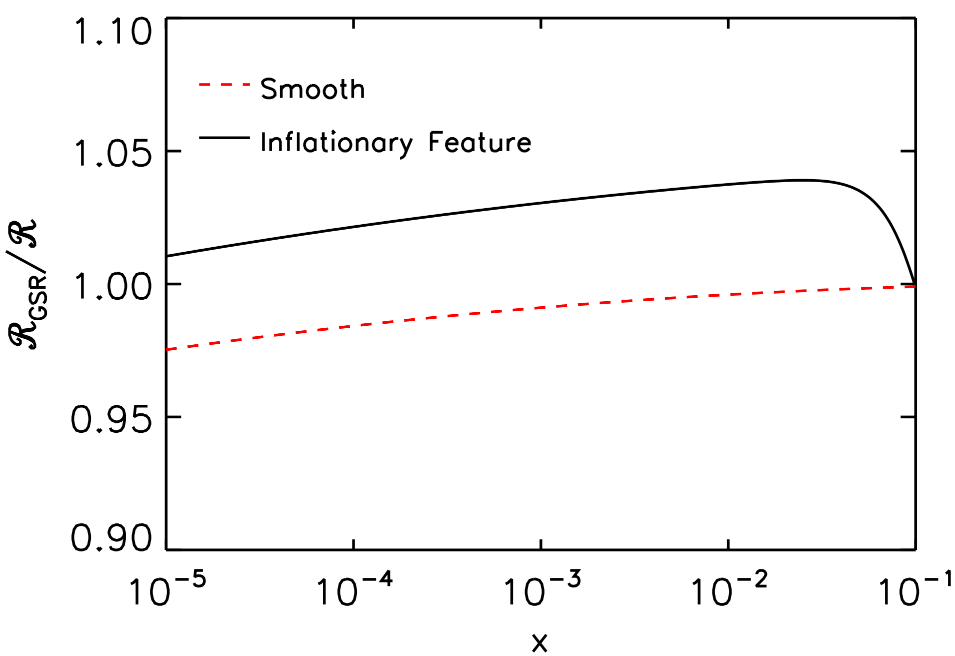
- Field equation: $$\frac{d^2y}{dx^2} + \left(1 - \frac{2}{x^2}\right) y = \frac{g(\ln x)}{x^2} y$$
  $(y = \sqrt{2k}u_k;\ x = k\eta)$
  Source function
- de-Sitter solution: $$\frac{d^2y_0}{dx^2} + \left(1 - \frac{2}{x^2}\right) y_0 = 0$$
- GSR approximation: $$\frac{d^2y}{dx^2} + \left(1 - \frac{2}{x^2}\right) y = \frac{g(\ln x)}{x^2} y_0$$

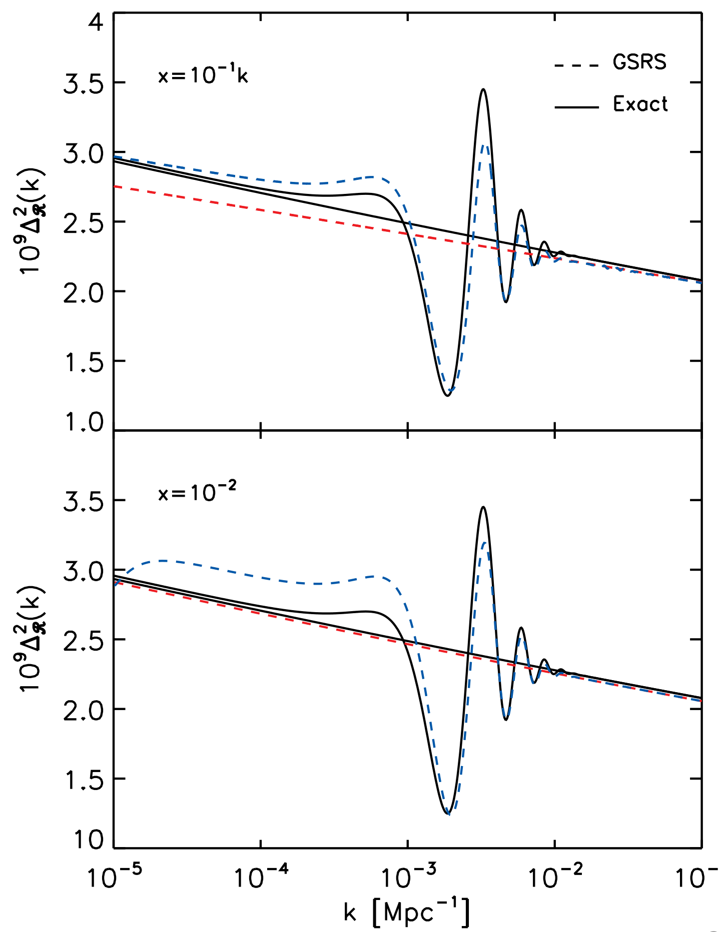
Solution can be constructed with a Green function approach:

$$y(x) \approx y_0(x) - \int_x^\infty \frac{du}{u^2} g(\ln u) y_0(u) \mathrm{Im}[y_0^*(u) y_0(x)]$$

Superhorizon evolution

Main problem: curvature is not constant for modes outside the horizon.





C.Dvorkin, W.Hu, PRD (2009)

Our GSR solution for large features

- The curvature power spectrum depends on a single source function:

$$\ln \Delta^2_{\mathcal{R}}(k) = G(\ln \eta_{\min}) + \int_{\eta_{\min}}^{\eta_{\max}} \frac{d\eta}{\eta} W(k\eta) G'(\ln \eta) + \ln \left[1 + \frac{1}{2} \left(\int_{\eta_{\min}}^{\eta_{\max}} \frac{d\eta}{\eta} X(k\eta) G'(\ln \eta) \right)^2 \right]$$

C.Dvorkin, W.Hu, PRD (2009)

- Source function on deviations from scale-invariance:

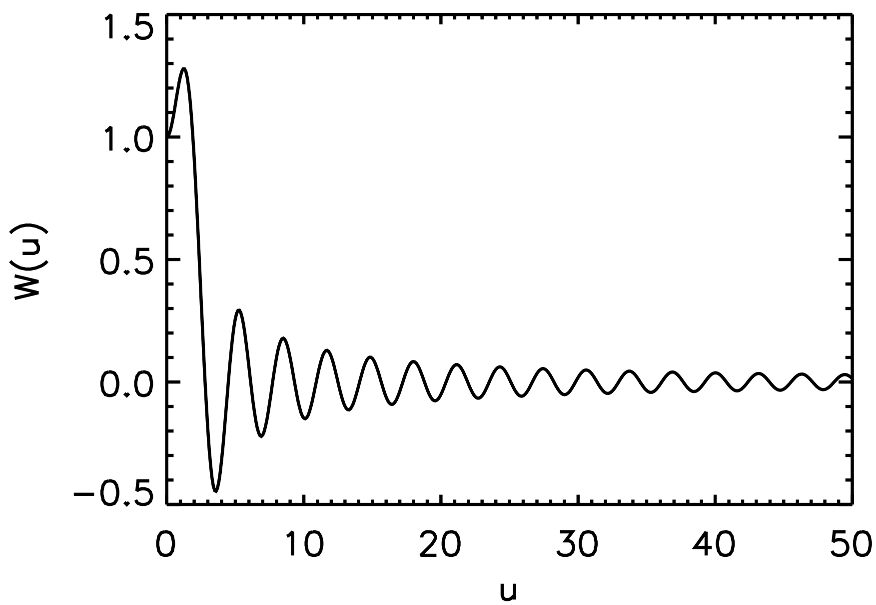
$$G' = \frac{2}{3} \left[\frac{f''}{f} - 3\frac{f'}{f} - \left(\frac{f'}{f}\right)^2 \right]$$ with $$f = 2\pi\eta \frac{\dot{\phi}}{H}$$

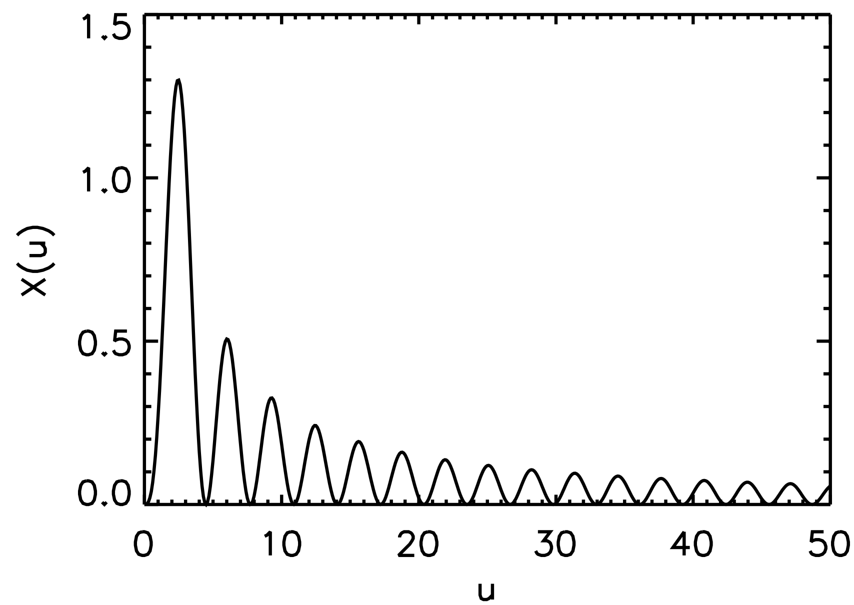
$\bullet = d/dt$

$' = d/d\ln\eta$

Second order correction to the source

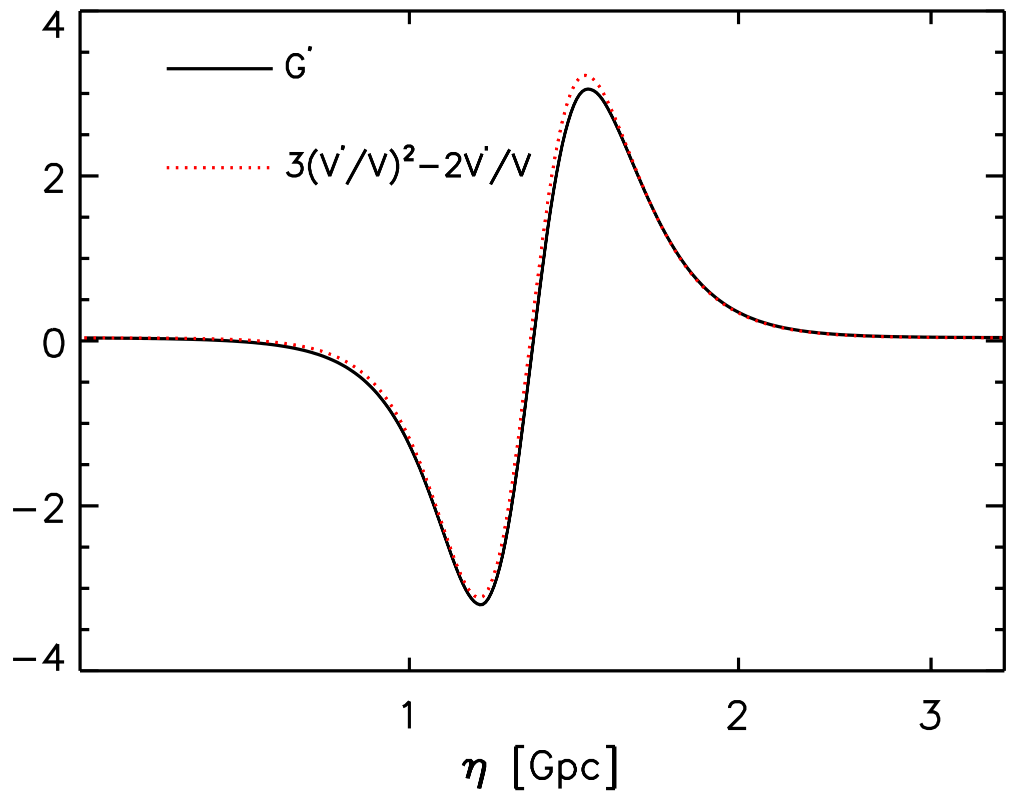
GSR Green functions





The source function and the Potential

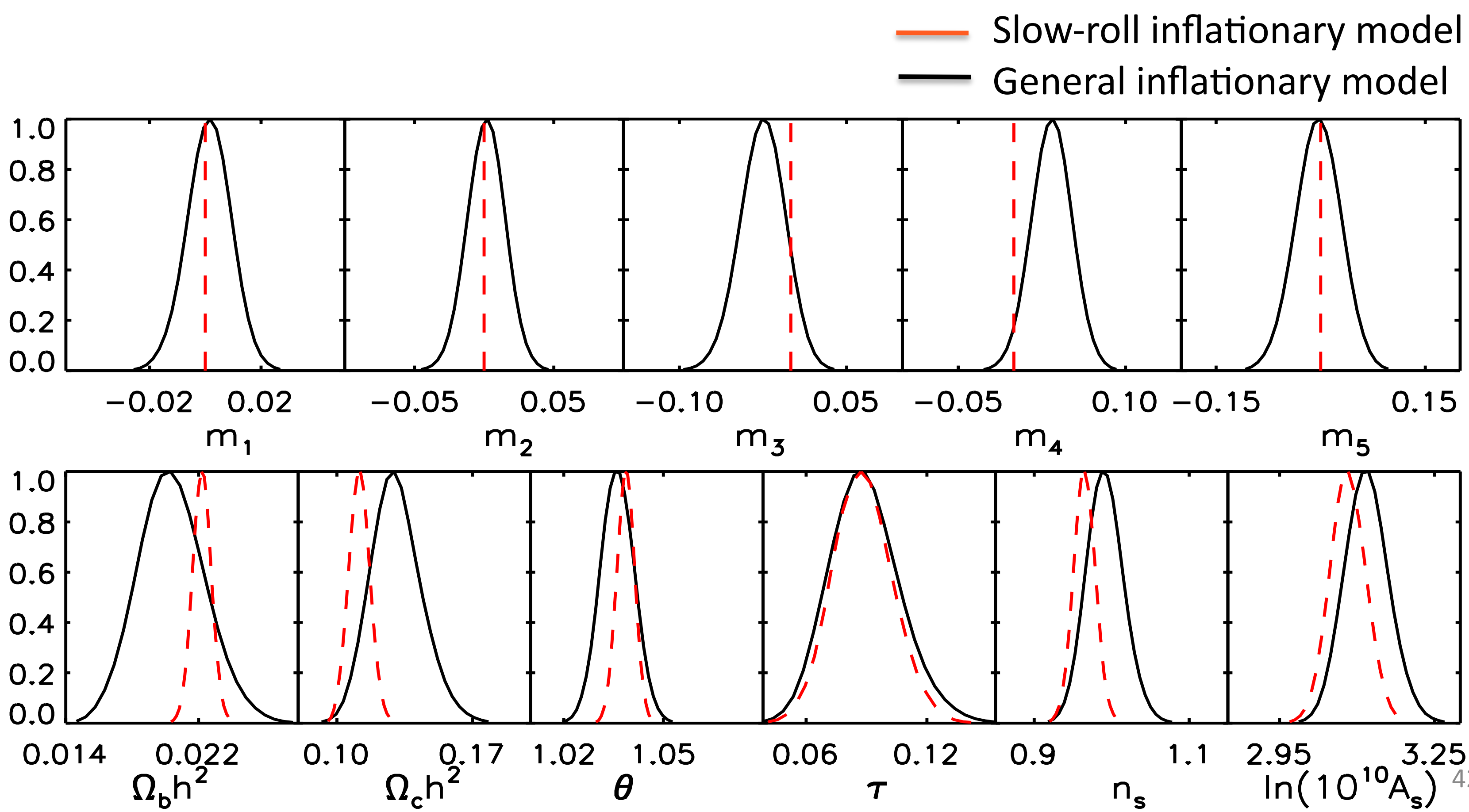
- Same functional dependence on the potential as the tilt in standard slow roll if features are crossed for an e-fold or less.
- Source has information on deviations from de-Sitter solution.



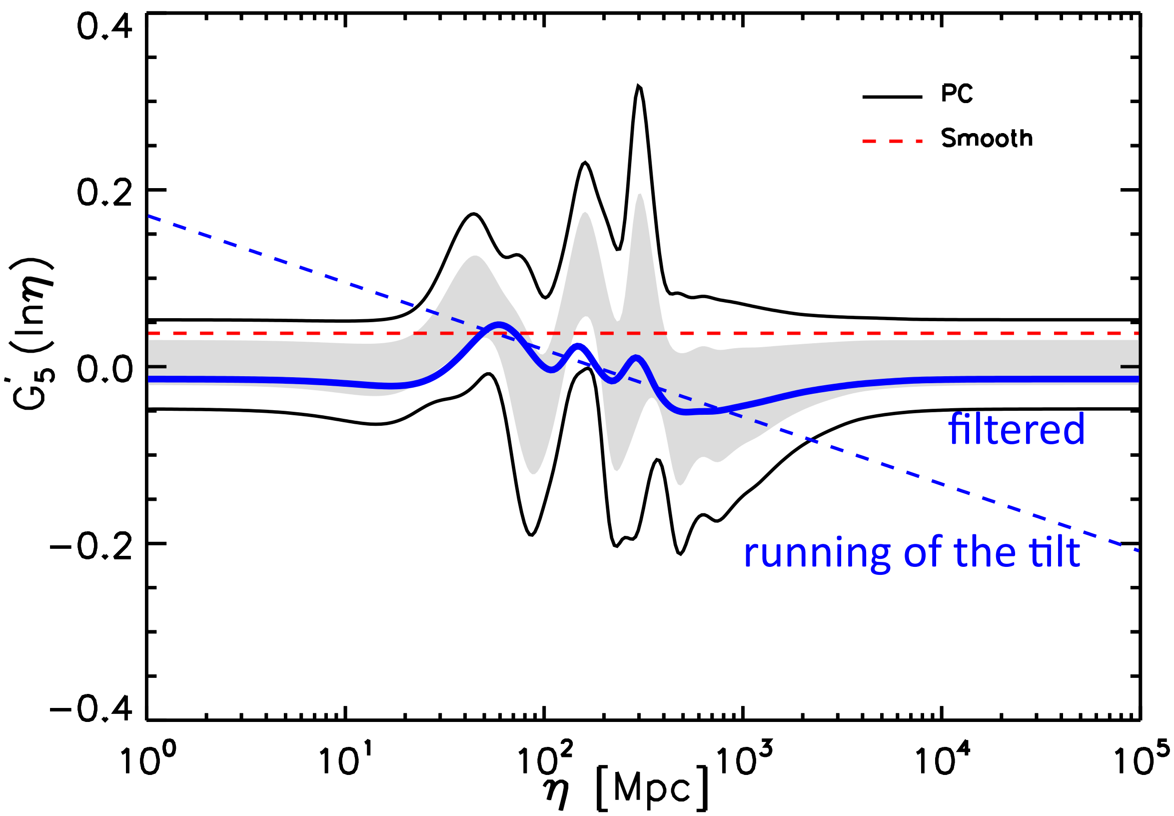
C. Dvorkin and W. Hu, PRD (2010)

WMAP7 constraints on the first 5 PCs

- The 4th component carries most of the information about running of the tilt.
- It resembles a local running of the tilt for $\ell \sim 30-800$, but it is marginally consistent with a constant running beyond this range.



Constraints on the source function with 5 PCs

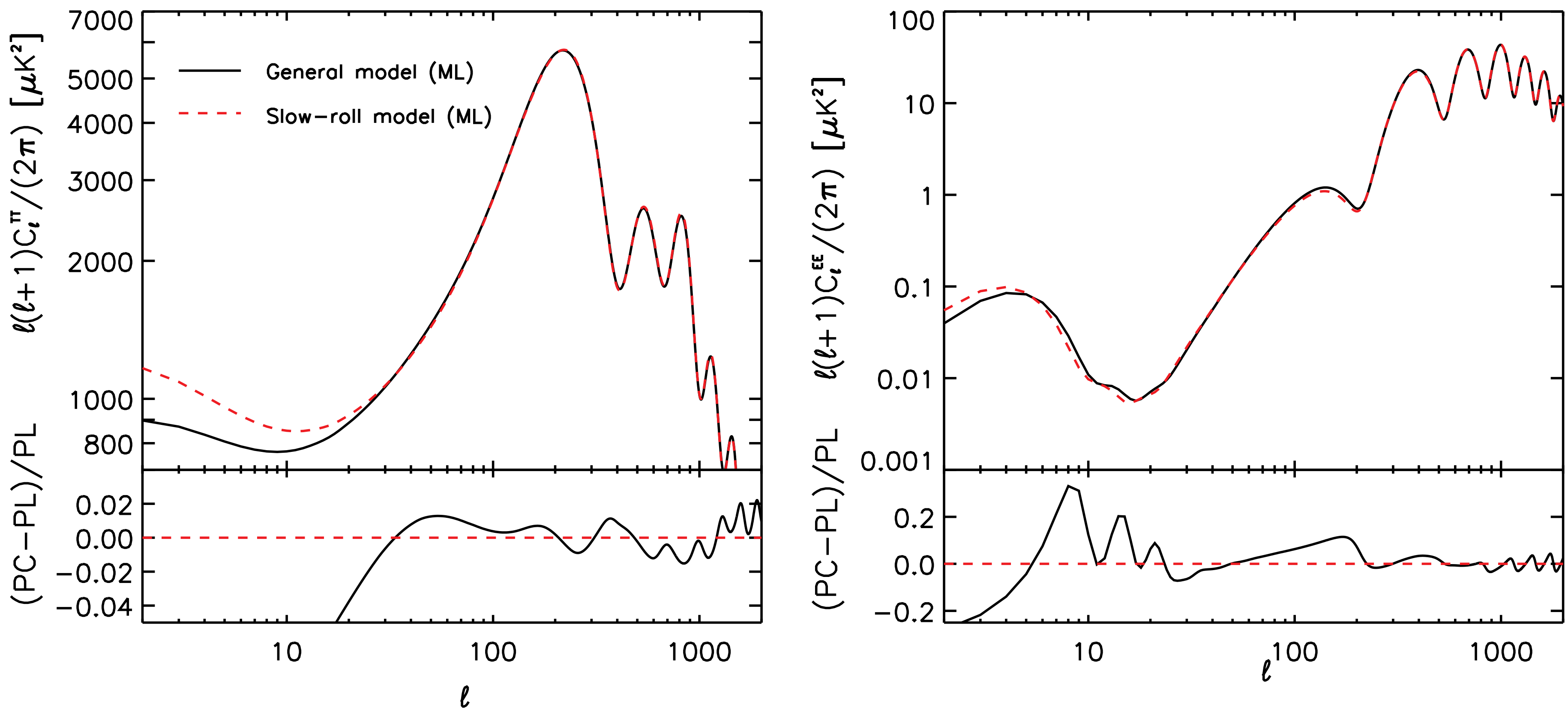


WMAP7, BICEP, QUAD;
SN, H0, BBN constraints;
flat universe.

C. Dvorkin, W. Hu, PRD (2009)

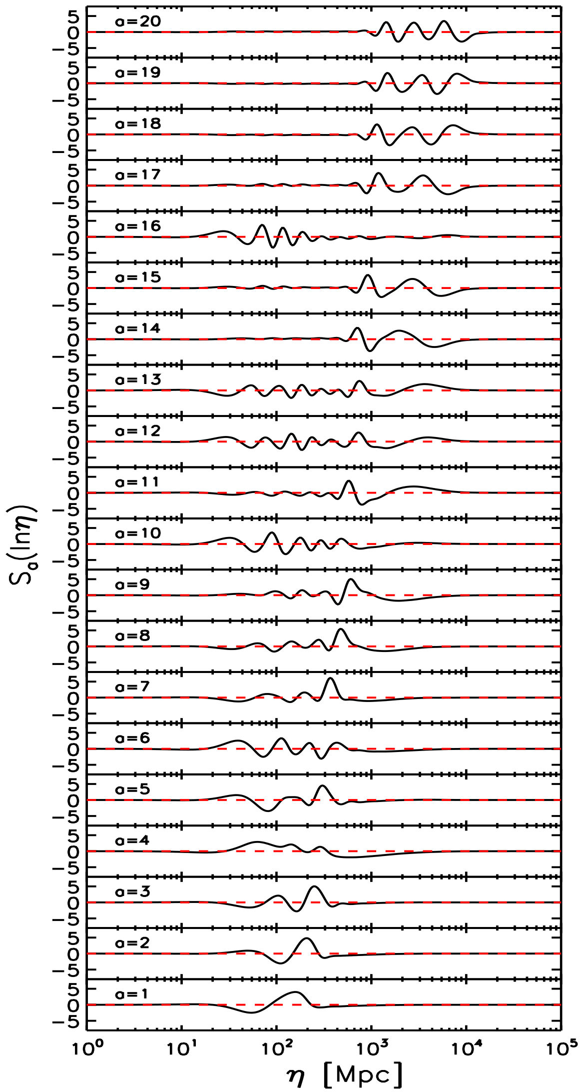
Future data: better constraints!

- Small-scale temperature measurements at $\ell > 1000$ and future polarization data at better than 10% at $\ell > 100$ (Planck) will improve inflationary constraints.



C.Dvorkin, W.Hu, PRD (2009)

Complete basis for Inflationary Features



- Complete basis for describing inflationary features that vary no more rapidly than 10 per decade in η.
- Features at low multipoles ($\eta = [10^3 - 10^4]$ Mpc) are represented by higher components: $S_{11} - S_{20}$

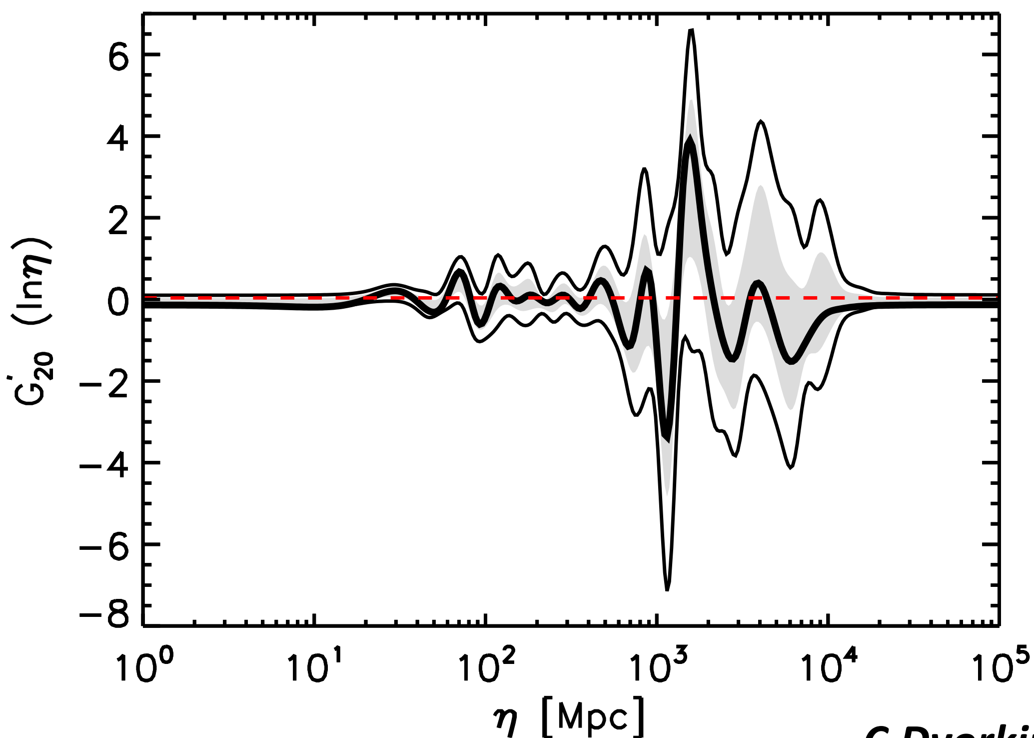
C.Dvorkin, W.Hu, PRD (2011)

Model-independent test of Slow Roll

- Constraints on G' impose constraints on features in the inflationary potential:

$$G' \approx 3\left(\frac{V_{,\phi}}{V}\right)^2 - 2\left(\frac{V_{,\phi\phi}}{V}\right)$$

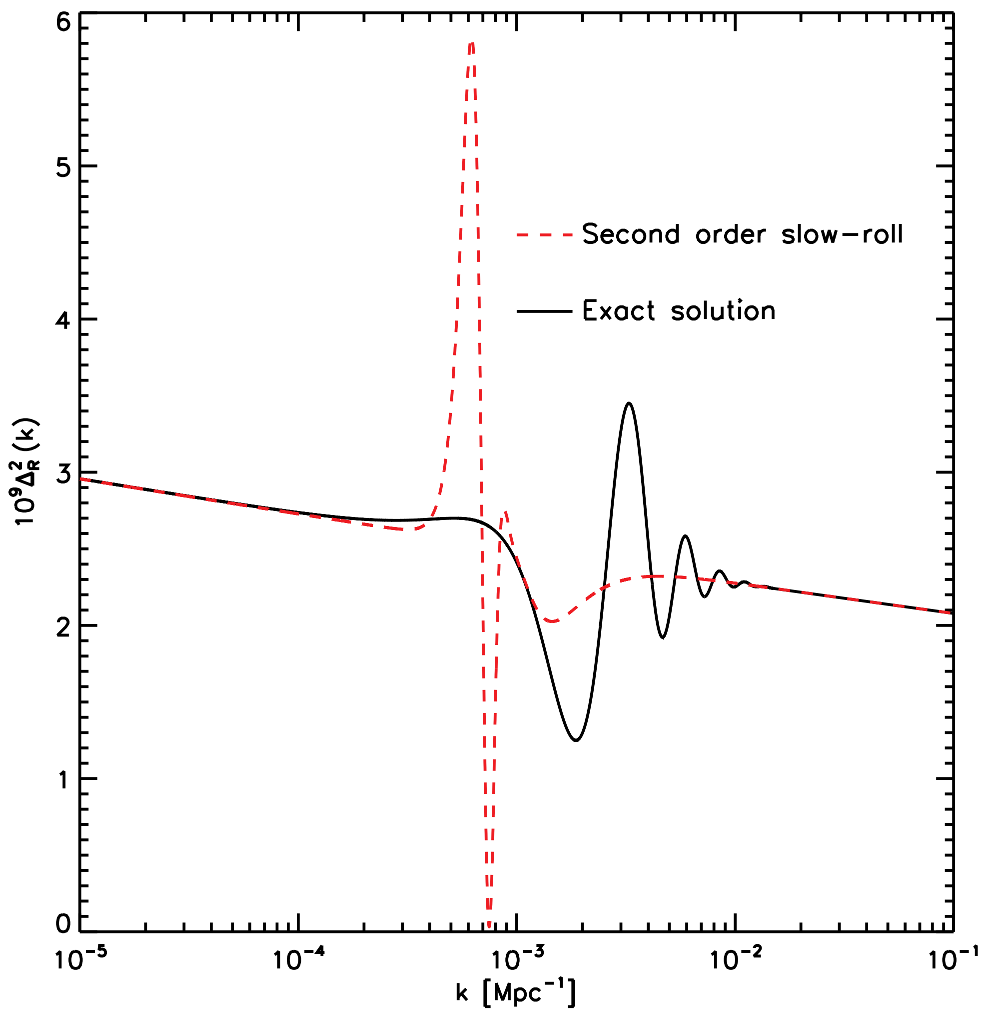
- Deviations from zero would indicate a violation of slow roll.



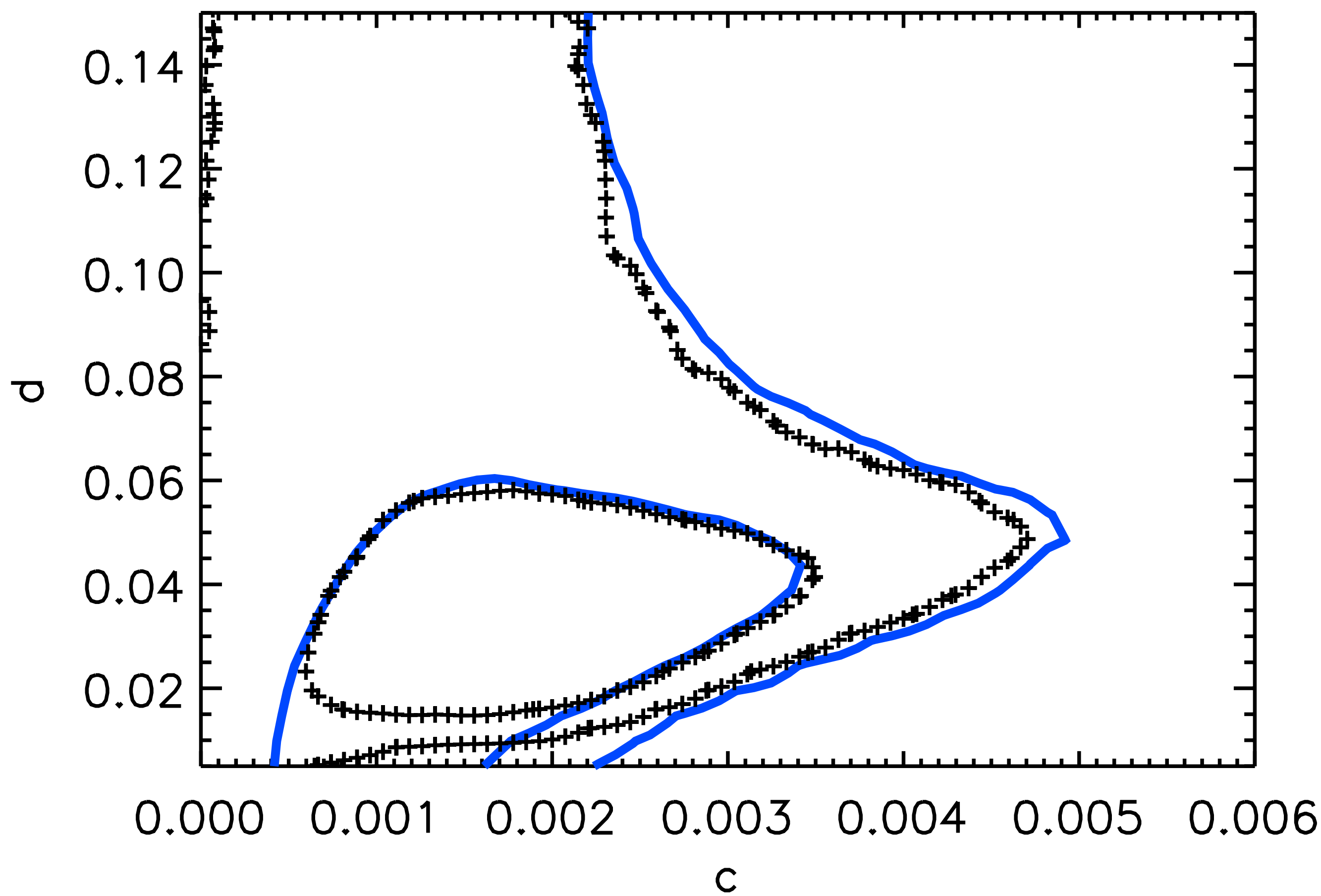
WMAP7 + BICEP + QUAD data;
SN + H0 + BBN constraints.

C.Dvorkin, W.Hu, PRD (2011)

Breaking Slow Roll



20 PCs vs. full GSR



Markov Chain Monte Carlo technique

- General method for efficiently obtaining constraints on parameters $\{\theta_1, \theta_2, ..., \theta_N\}$ given a probability distribution $P(\theta_1, \theta_2, ..., \theta_N)$

- Metropolis algorithm moves from position in parameter space $\vec{\theta}$ to $\vec{\theta'}$ with transition probability:

$$T(\vec{\theta}, \vec{\theta'}) = min\{1, \frac{P(\vec{\theta'})}{P(\vec{\theta})}\}q(\vec{\theta}, \vec{\theta'})$$

Proposal density

- This choice of transition probability ensures that the Markov chain has a stationary asymptotic probability distribution:

$$P(\vec{\theta'})T(\vec{\theta'}, \vec{\theta}) = P(\vec{\theta})T(\vec{\theta}, \vec{\theta'})$$

MCMC optimizations

- Likelihood corrections:
  - Low-ℓ polarization approximation
  - Lensing off
  - Low-ℓ sampling
- Thinning
  (samples are highly correlated)
- Post-process (parallel)
- Change in parameterization
  (to reduce parameter degeneracies):
  - Tilt averaged over a narrower range in η
  - Normalization in ℓ-space