Internal wave breaking and the fate of planets around solar-type stars

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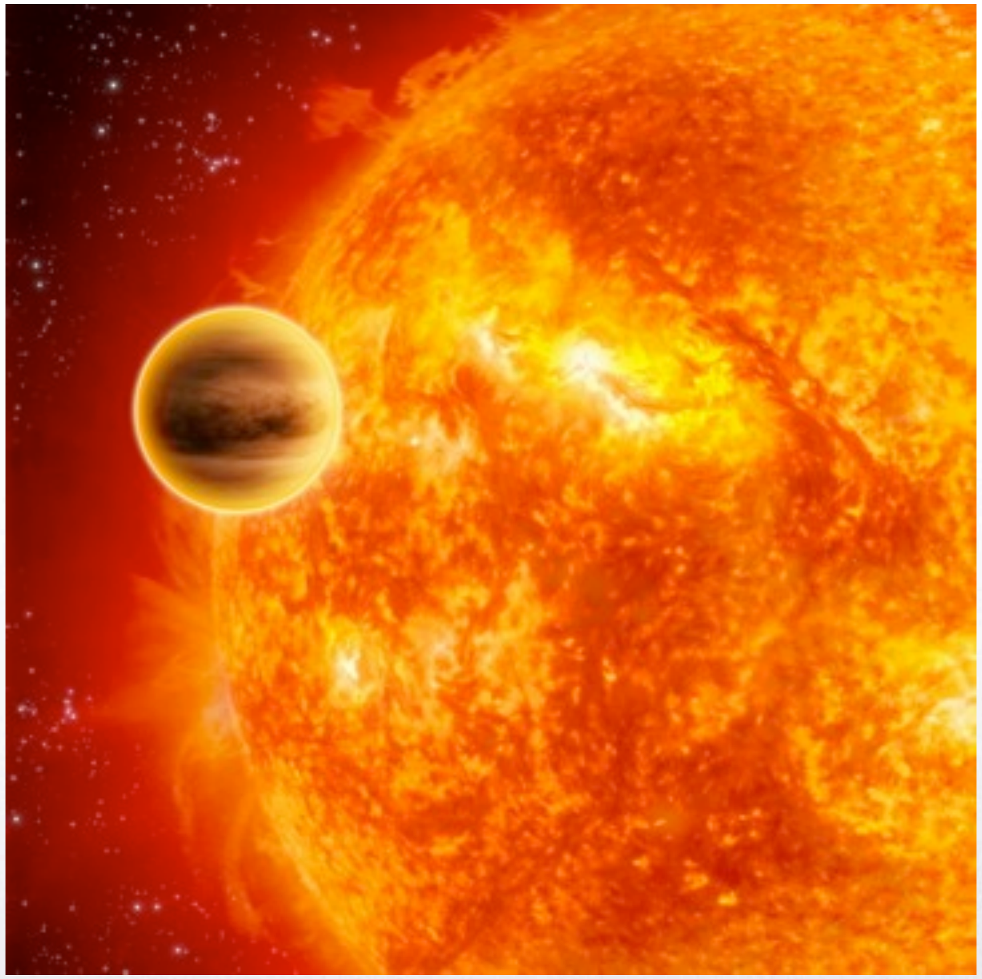
Hot Jupiters

- Since 1995, we have detected nearly 600 extrasolar planets
- ~1/6 of these are Hot Jupiters
- ~35 HJs orbit G or K type stars in orbits with periods < 3 days
- The closest (WASP-19 b) has an orbit of 0.78 days!

Can we explain their survival against tidal decay?



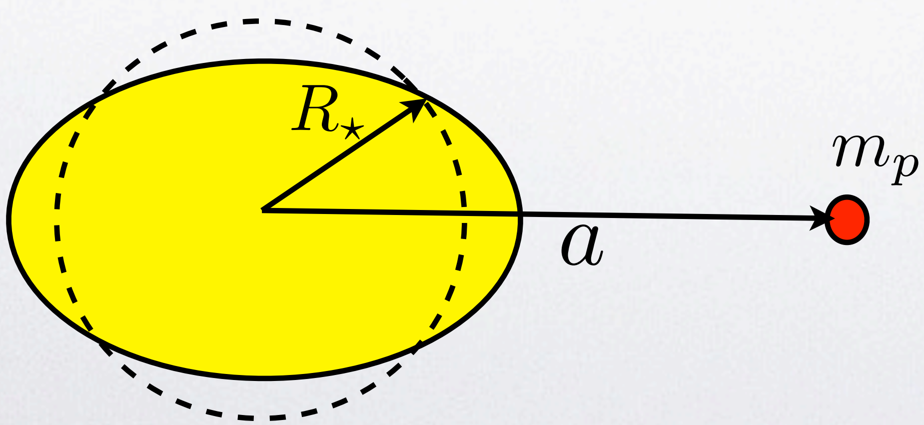
Tidal response in star

- Tidal potential, at radius r from centre of star, caused by a planet on circular, coplanar orbit is

$$\Psi = A\frac{Gm_p}{a^3}r^2P_2^2(\cos\theta)\cos(2\phi - \omega t)$$

- Dissipation in star & planet causes spin-orbit evolution

- If $P < P_\star$, dissipation in star causes planetary inspiral, produced by the torque



$$|T| \propto \frac{1}{Q'}\frac{Gm_p^2R_\star^5}{a^6}A^2$$

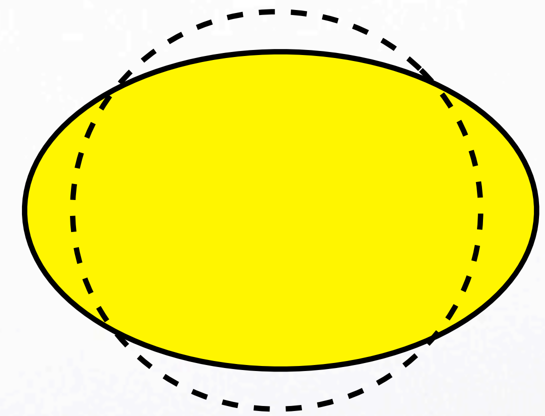
$$Q' \propto E_0\left(\oint \dot{E}\mathrm{dt}\right)^{-1}$$

Tidal response in star

The response of a fluid body to tidal forcing can be decomposed into two parts:

(1) **Equilibrium tide** (e.g. Darwin, 1880)

quasi-hydrostatic tidal bulge, dragged around the body



damped by turbulent convection in convection zones (e.g. Zahn, 1966; ...)

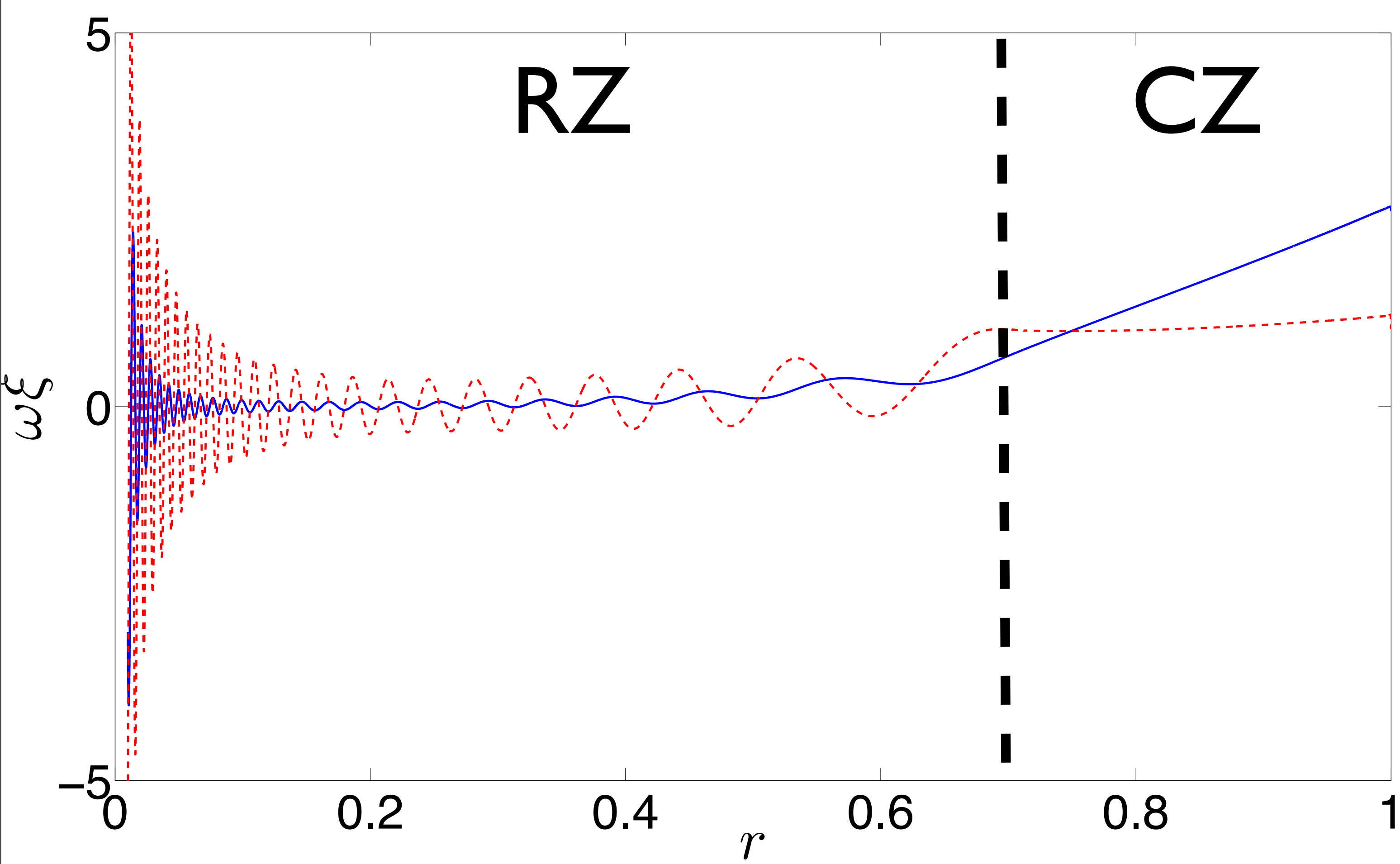
(2) **Dynamical tide**

additional wavelike response, takes the form of:

inertial waves in convective regions (e.g. Ogilvie & Lin, 2004,2007; Wu 2005; Papaloizou & Ivanov 2010)

inertia-gravity waves in radiative regions (e.g. Zahn 1970; Goldreich & Nicholson 1989; Goodman & Dickson 1998)

Tidal response in star



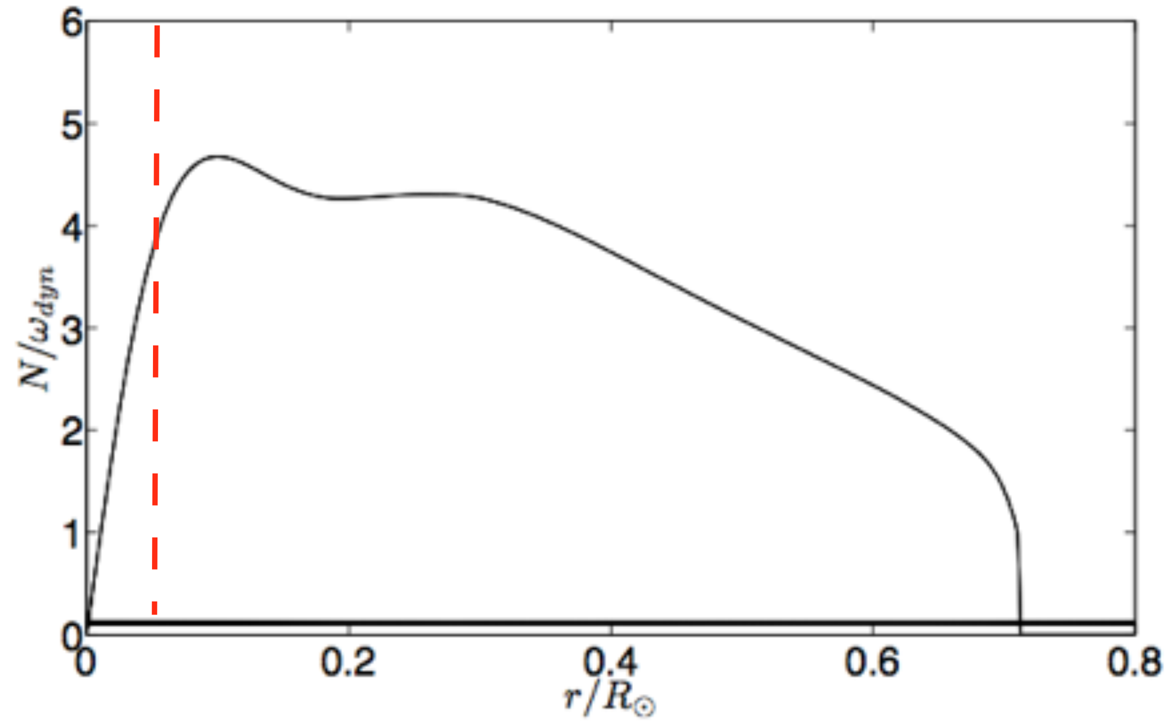
$$\omega^2 = \frac{k_h^2}{k^2} N^2$$

- Linearised tidal response in a model of the current Sun
- In the RZ, the response is oscillatory, with the magnitude of the oscillations increasing near the centre.
- Tidal forcing by a HJ excites internal gravity waves (IGWs) at the top of the RZ of a (non-rotating) solar-type star
- IGWs propagate towards the centre and their dissipation contributes to Q'
  Goodman & Dickson, 1998
  Terquem et al., 1998

Central regions

- In the innermost regions

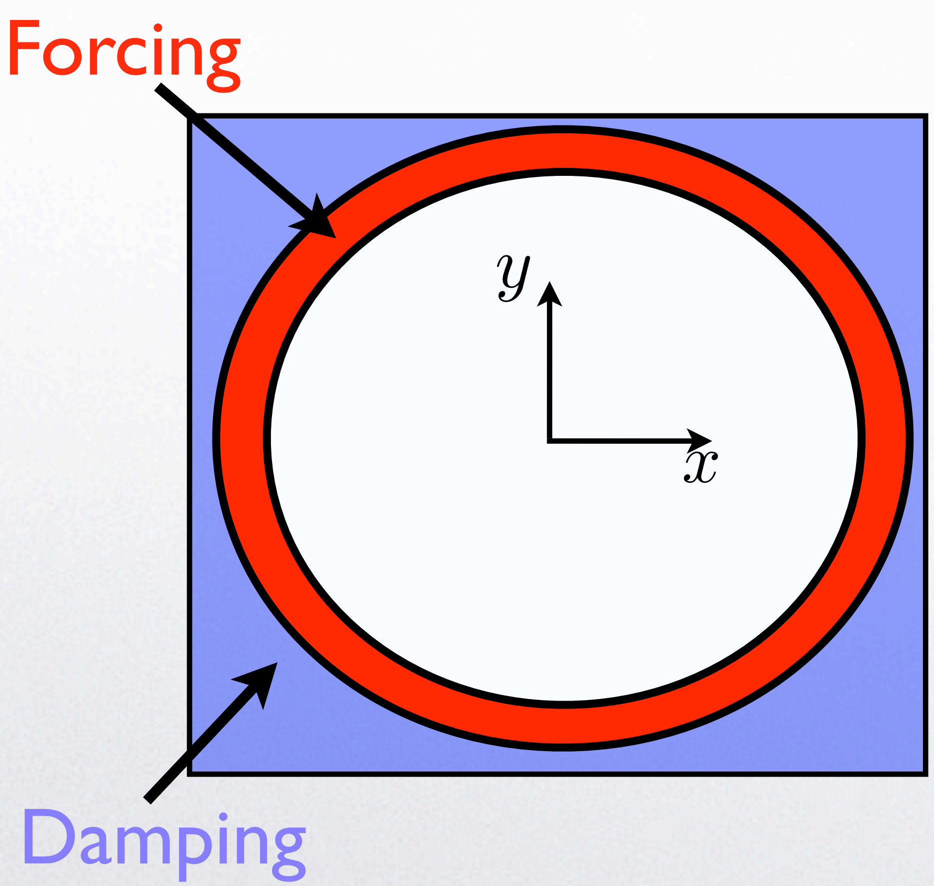
$$\begin{aligned} N &= Cr, \\ g &\propto r, \end{aligned}$$



- A Boussinesq-type system has been derived for this region, with one parameter, C

$$\begin{aligned} D\mathbf{u} &= -\nabla q + \mathbf{r}b, \\ Db + C^2\mathbf{r}\cdot\mathbf{u} &= 0, \\ \nabla\cdot\mathbf{u} &= 0, \\ D &= \partial_t + \mathbf{u}\cdot\nabla. \end{aligned}$$

Numerical methods



Model

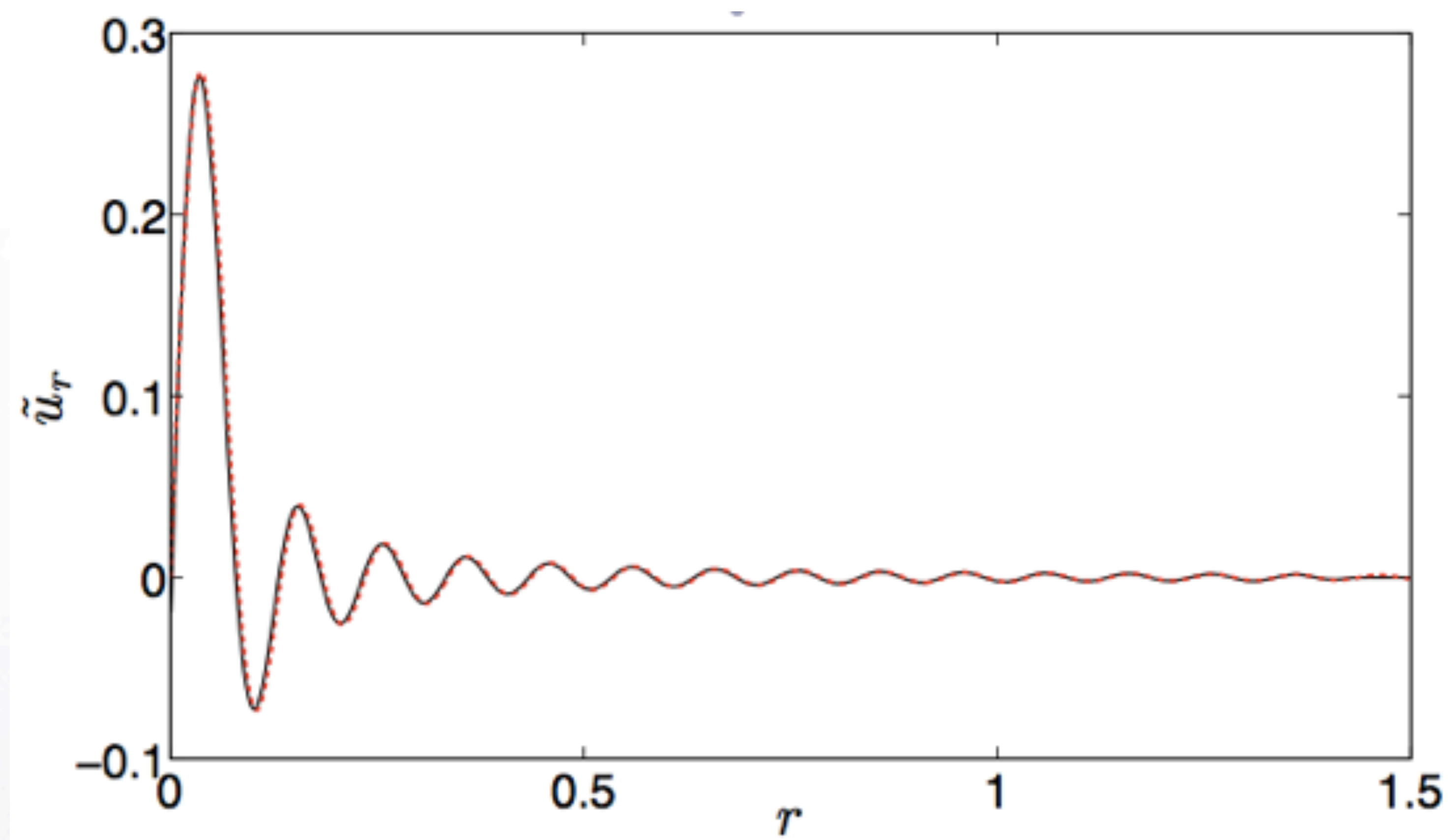
- We use SNOOPY, a Cartesian spectral code (G. Lesur)
- Non-rotating, 2D, circularly symmetric star (or 3D, spherically symmetric)
- Apply forcing to radial momentum equation to excite waves, relevant for planets on circular, coplanar orbits $\propto \cos(2\phi - \omega t)$
- Smooth solutions to zero in the outer regions to satisfy periodic BCs in SNOOPY

Wave solution

- Response steady in frame rotating with the planetary orbit Ω_p
- Find a wave solution, in dimensionless units where $[T] = \Omega_p^{-1}$, $[L] = \dfrac{\Omega_p}{C}$

$$u_r \propto \frac{A}{r} J_2(r)$$

This is an exact (nonlinear) solution in 2D!



Comparison of SNOOPY simulation with analytic result for low-amplitude forcing.

Low-amplitude vs high-amplitude forcing

$A < 1$ $N^2 \geq 0$

- Low-amplitude forcing => (approximately) perfect reflection from the centre
- Global standing modes can form in radiation zone
- No instability is observed to act on waves

Do any instabilities exist in reality? Implications...

$A > 1$ $N^2 < 0$

- High-amplitude forcing => convective instabilities
- Leads to wave breaking => critical layer (corotation radius) forms at the centre, where the fluid angular velocity is Ω_p
- Ingoing wave angular momentum absorbed at the centre of the star => strong tidal torque

Conclusions

- Short-period planets cause wave breaking at the centre of a star with a radiative core if the waves that they excite overturn the stratification. This occurs around the current Sun if

$$\left(\frac{C}{C_\odot}\right)^{\frac{5}{2}}\left(\frac{m_p}{M_J}\right)\left(\frac{P}{1\,\mathrm{day}}\right)^{\frac{1}{6}} \gtrsim 3.3$$

- The resulting dissipation is efficient with $Q' \approx 1.5 \times 10^5 \left[\frac{P}{1\,\mathrm{day}}\right]^{\frac{8}{3}}$

  This does not vary more than a factor of 5 between all main-sequence stars in the range $0.5 \leq m_\star/M_\odot \leq 1.1$, for a given orbit.

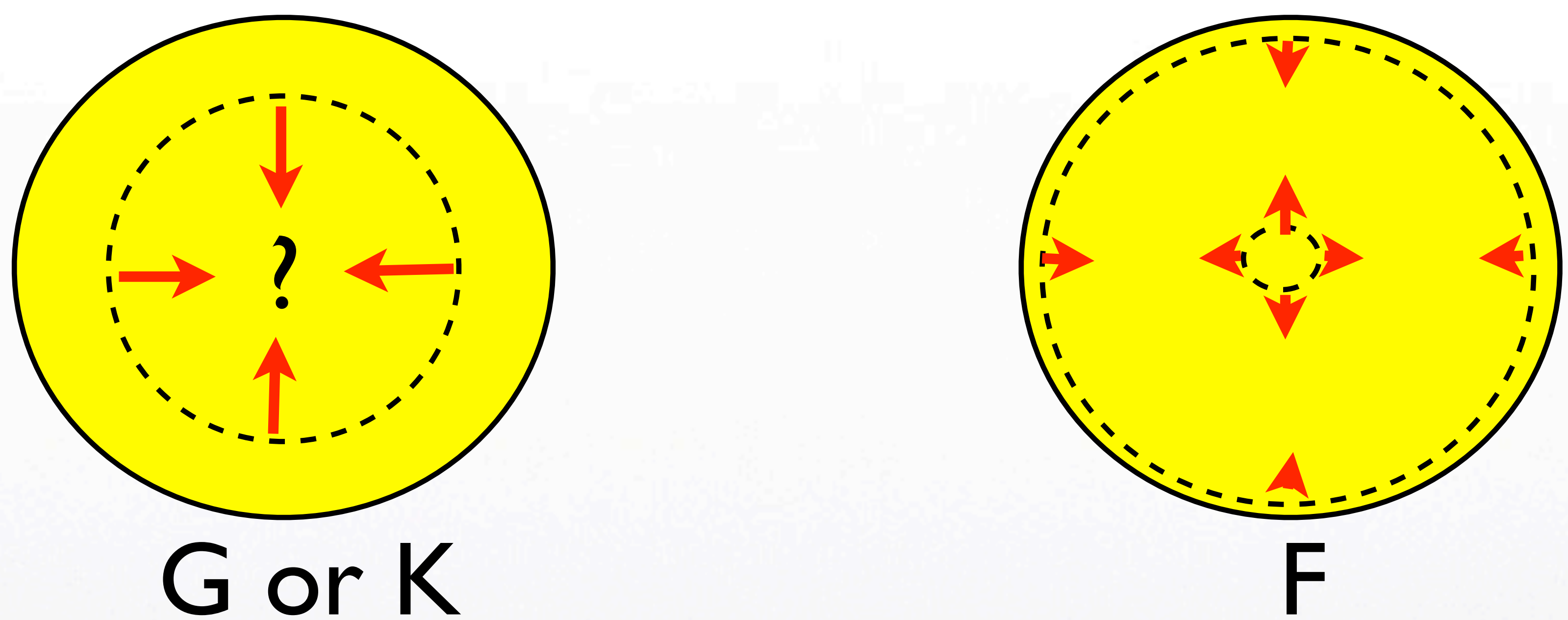
- Leads to an inspiral time ~ Myr < Gyr for a HJ on an orbit with P < 2-3 d

$$\tau_a \approx 2\,\mathrm{Myr}\left(\frac{M_J}{m_p}\right)\left(\frac{P}{1\,\mathrm{day}}\right)^7$$

- *The absence of wave breaking can explain the survival of all short-period planets around F, G and K stars that have been observed thus far, and will be tested in future observations.*

Conclusions II

- The absence of wave breaking can explain the survival of all short-period planets around G and K stars that have been observed thus far, and will be tested by future observations.



- It can also partially explain why the most massive short-period planets are found around F-stars, with convective cores (IGWs are reflected from CZ/RZ interfaces, so don't reach centre with high amplitudes)

Stability analysis of wave

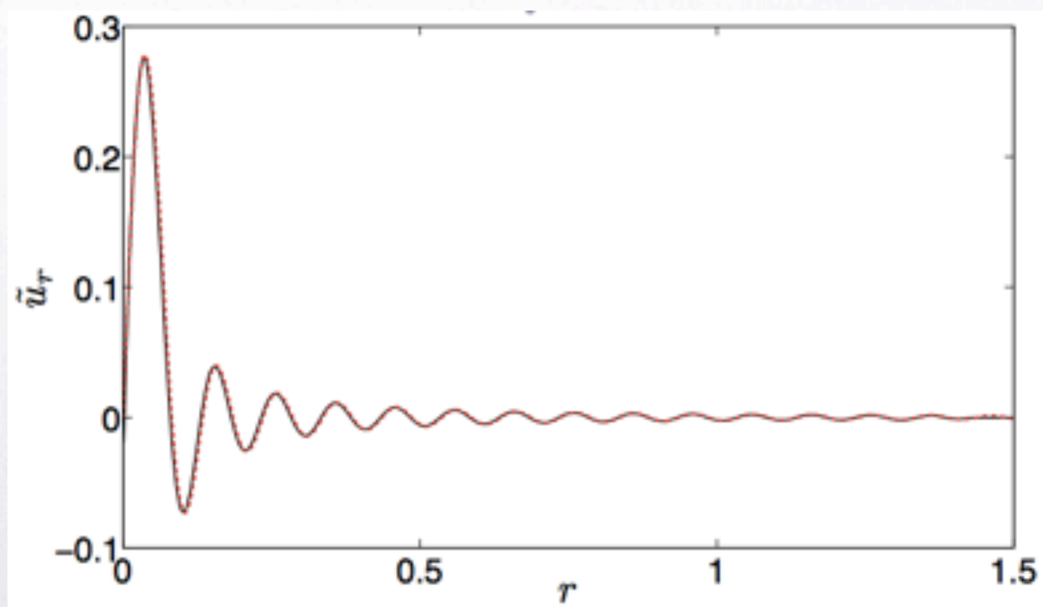
Questions:

1. Do any instabilities exist for low-amplitude waves (excited by low-mass planets)?

2. Can we better understand the breaking process for high-amplitude waves?

Method: Perform stability analysis of primary wave, along the lines of stability analyses for plane IGWs in uniform stratification e.g. Drazin 1977, Klostermeyer 1982, ...

- Start with the primary wave and look for small perturbations to the wave



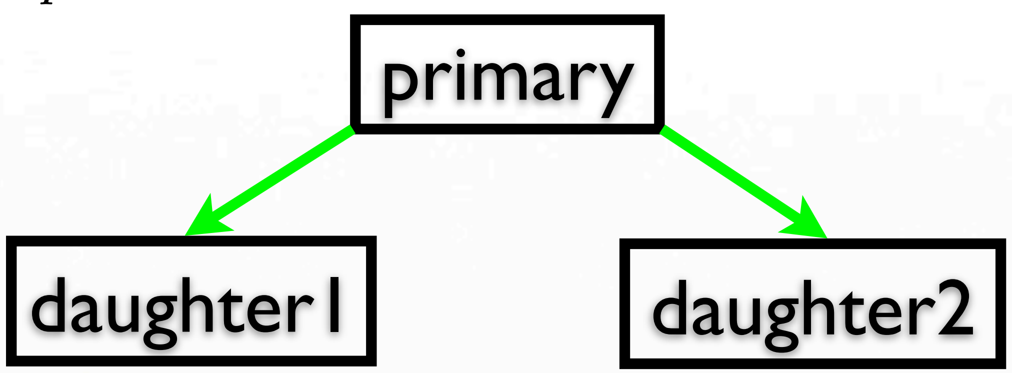
by writing

$$b = b_w + b'$$
$$\psi = \psi_w + \psi'$$

Low-amplitude vs high-amplitude

$A < 1$

- Parametric instabilities exist for any amplitude: two daughter modes satisfying $\omega_p \approx \omega_{d1} + \omega_{d2}$



- Driven by free energy associated with primary wave (stable) entropy gradients (not shear)
- *Parametric instabilities are important only in a small domain* since $\mathrm{Im}\,[\omega] \sim 1/n_p$
- Leads to $Q'_\star \gtrsim 10^7$ (at least)

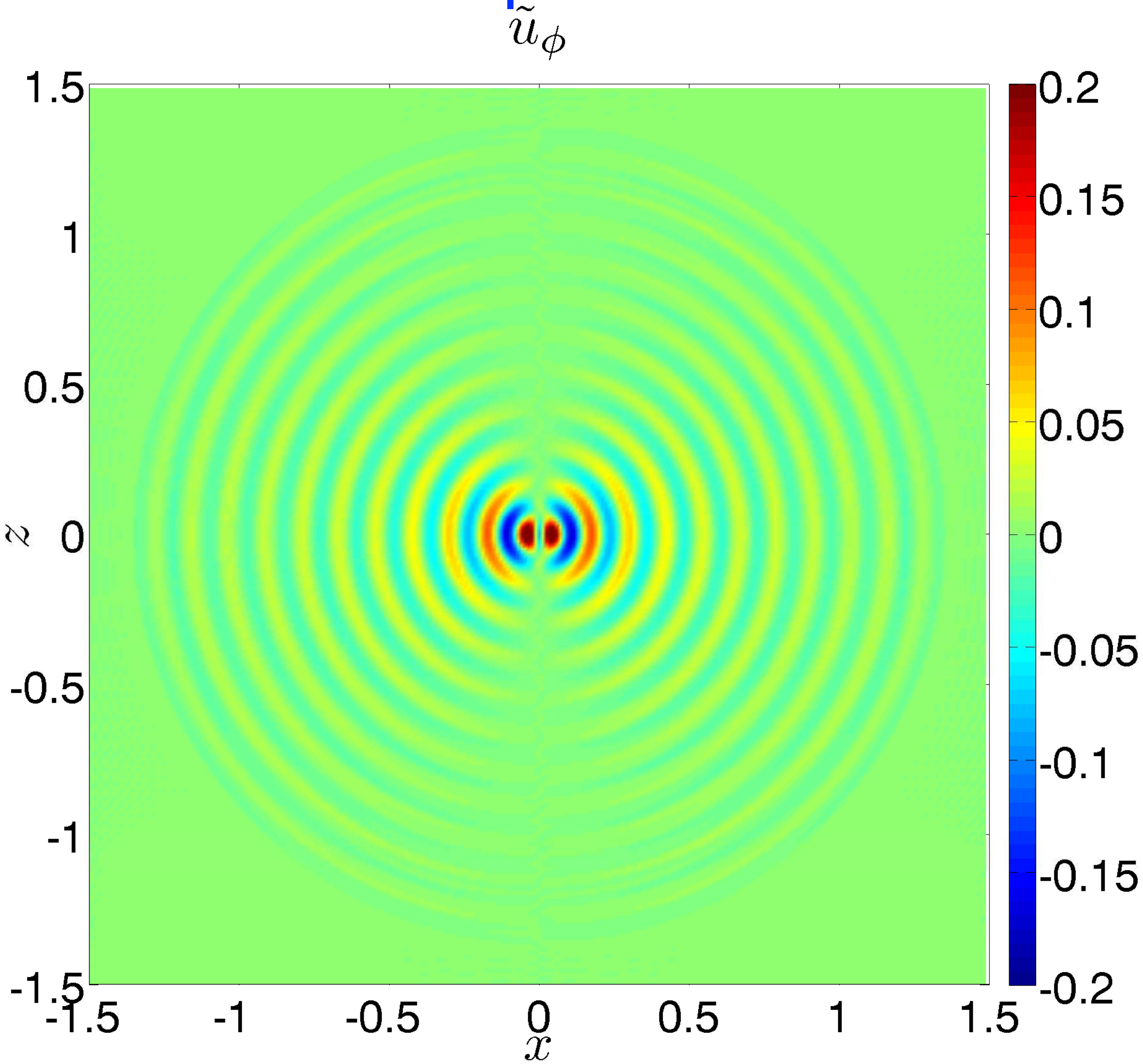
$A > 1$

- Instability that breaks the wave is strongly localised in convectively unstable regions
- Driven by free energy associated with primary wave (unstable) entropy gradients (not shear)
- Rapidly grow c.f. primary wave period i.e. $\mathrm{Im}\,[\omega] \sim \omega_p$
- Nonlinear outcome leads to critical layer formation => ang mom absorption
- Leads to $Q' \approx 1.5 \times 10^5 \left[\frac{P}{1\,\mathrm{day}}\right]^{\frac{8}{3}}$

3D simulations

- Latitudinal differential rotation

Low-amplitude



$$\mathrm{Re}[Y_2^2(\theta,\phi)] \propto \sin^2\theta\cos(2\phi)$$

High-amplitude