Tidal dissipation in convective regions of planets and stars

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Effects of tidal dissipation

Dissipation in the star :

- orbital decay
- spin-orbit alignment

} in most cases

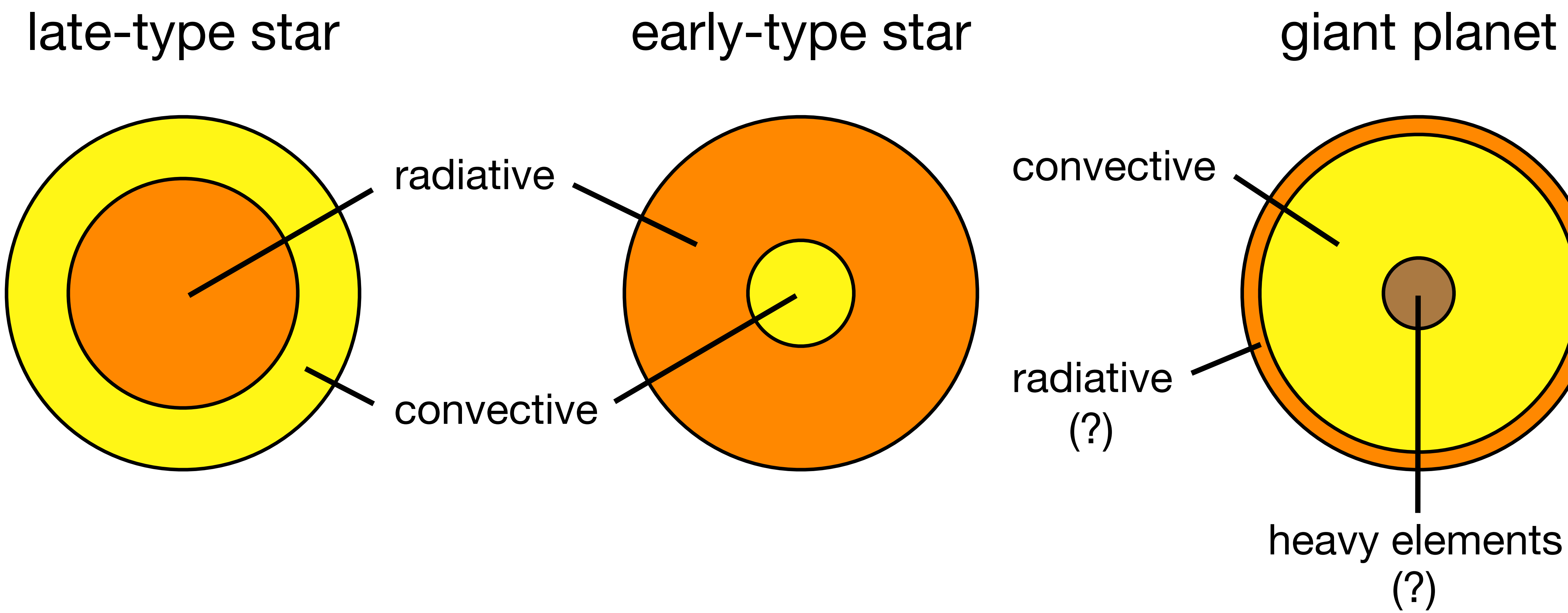
Dissipation in the planet :

- spin synchronization
- orbital circularization
- heating

Tidal theory :

- determine the rates of these processes
- how much dissipation (or torque) is produced when a body is forced by a potential $\propto r^l Y_l^m(\theta,\phi)\exp(-\mathrm{i}\omega t)$?
  (depends on frequency ω and quantum numbers l, m)

Tides in convective regions of planets and stars



- Turbulent viscosity acting on tidal bulge?
- Excitation and dissipation of inertial waves?

[For radiative regions see poster 34.03 by Adrian Barker]

Linear tides in barotropic fluid bodies

- Barotropic : no stable stratification or internal gravity waves
- Low-frequency tides in slowly rotating bodies :

$$\omega \sim \Omega \sim \epsilon \left(\frac{GM}{R^3} \right)^{1/2}, \quad \epsilon \ll 1$$

- Systematic theory based on expansion in powers of ϵ^2
- Displacement $\boldsymbol{\xi} = \boldsymbol{\xi}_{\mathrm{nw}} + \boldsymbol{\xi}_{\mathrm{w}}$
- Non-wavelike part :

  response of spherical body to tidal potential neglecting Coriolis

  (easily computed but different from classical equilibrium tide)
- Wavelike part :

  residual response (inertial waves)

  known body force from Coriolis force on non-wavelike part

Periodic forcing of inertial waves

- Consider inertial waves driven by body force $\propto \exp(-\mathrm{i}\omega t)$

  deriving from tidal potential $\propto r^l Y_l^m(\theta,\phi)\exp(-\mathrm{i}\omega t)$
- Calculate linear response with same frequency

Selected references :

Ogilvie & Lin (2004)

Wu (2005)

Ogilvie & Lin (2007)

Ivanov & Papaloizou (2007, 2010)

Goodman & Lackner (2009)

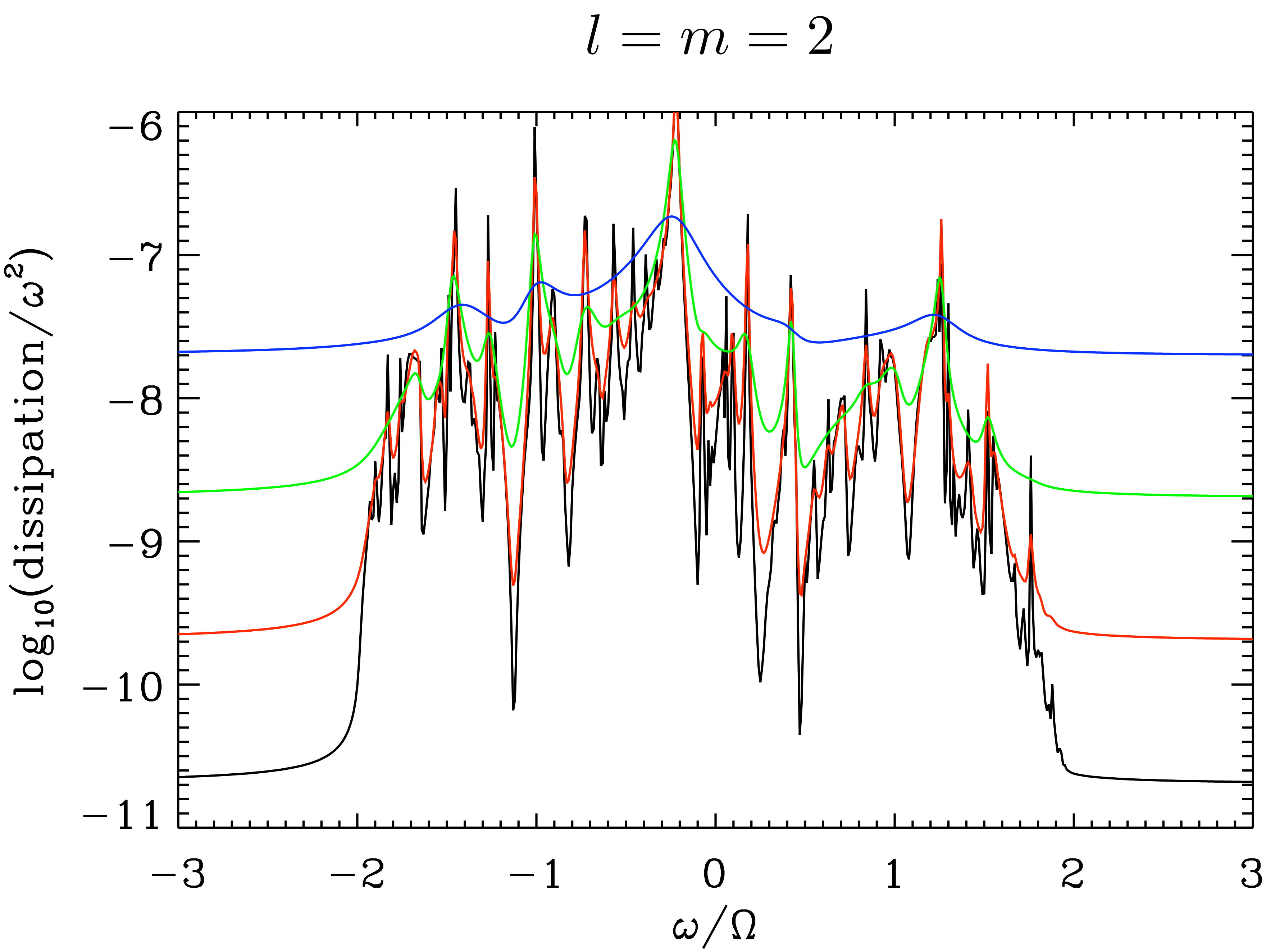
Ogilvie (2009)

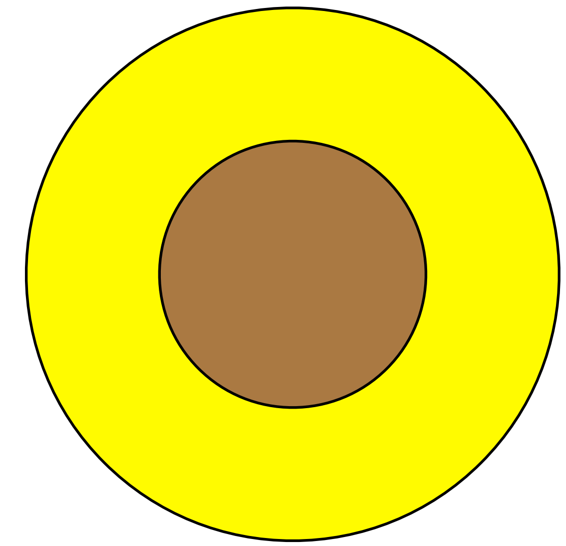
Rieutord & Valdettaro (2010)

Typical results

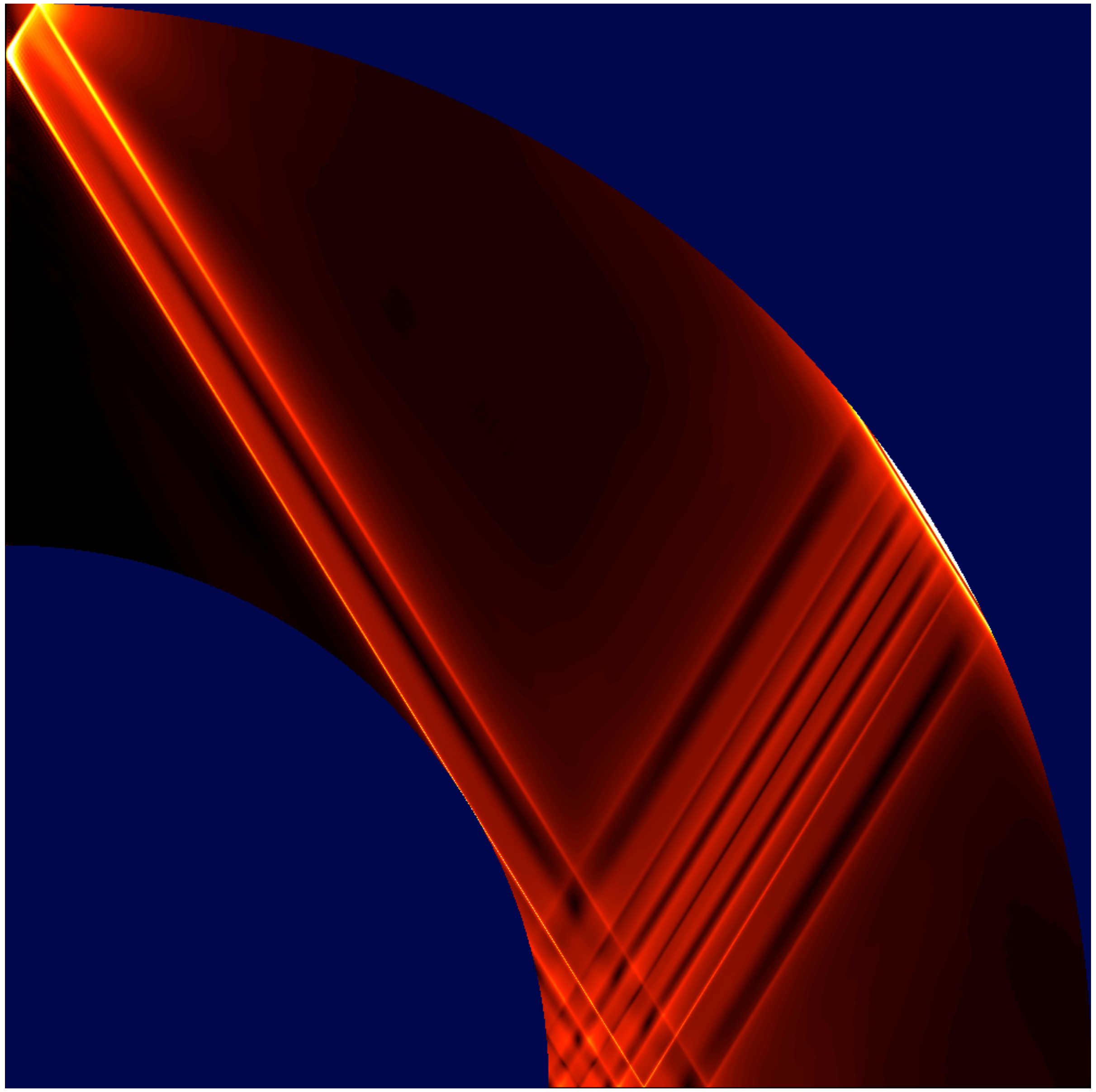
Idealized problem : isentropic rotating fluid in spherical geometry

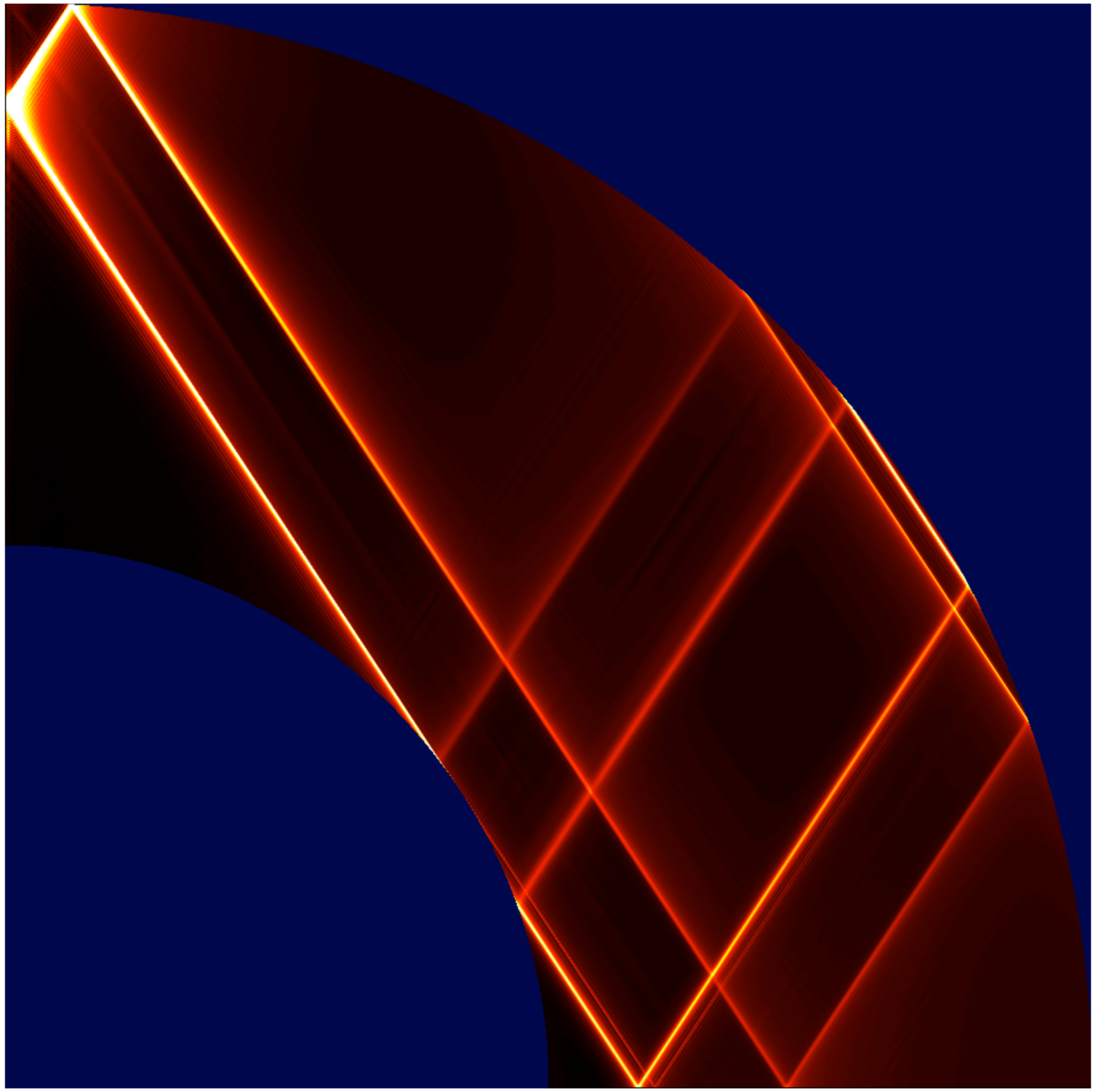
- Rigid core, fractional radius 0.5





Typical results





- Caveats :
  - convective background
  - magnetic fields
  - reflections
  - nonlinear breakdown

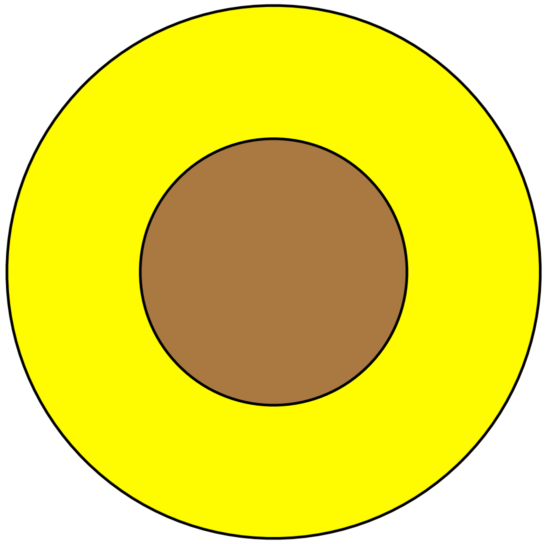
Impulsive forcing of inertial waves

- Consider inertial waves driven by body force $\propto \delta(t)$
- Impulsive response is smooth and readily computable (ODEs)
- Will subsequently resolve into complicated pattern of inertial waves which may dissipate through linear or nonlinear processes
- (Kinetic) energy of impulsive response gives a broad frequency average of the response function
- Equivalent to frequency-average of tidal time lag, or $\dfrac{1}{\omega Q}$
- Robust result, dependent only on gross structure
- Also more directly applicable to highly eccentric orbits

Impulsive energy transfer / frequency-averaged dissipation

- Homogeneous fluid body with rigid core
  - Sectoral harmonics $m = l$

$$\hat{E} \propto \frac{\alpha^{2l+1}}{1-\alpha^{2l+1}} \qquad \alpha = \frac{r_{\mathrm{in}}}{R}$$



  - Tesseral harmonics $m < l$

$$\hat{E} \propto \frac{1}{1-\alpha^{2l+1}} \qquad (+\ \text{term as above})$$

but beware trivial inertial modes with $l = 2$

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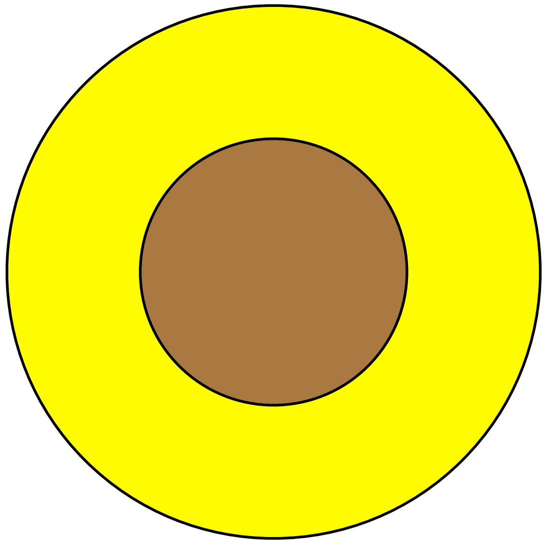

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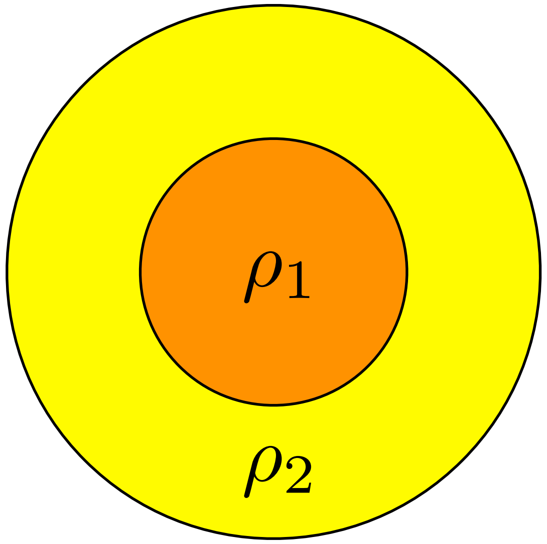
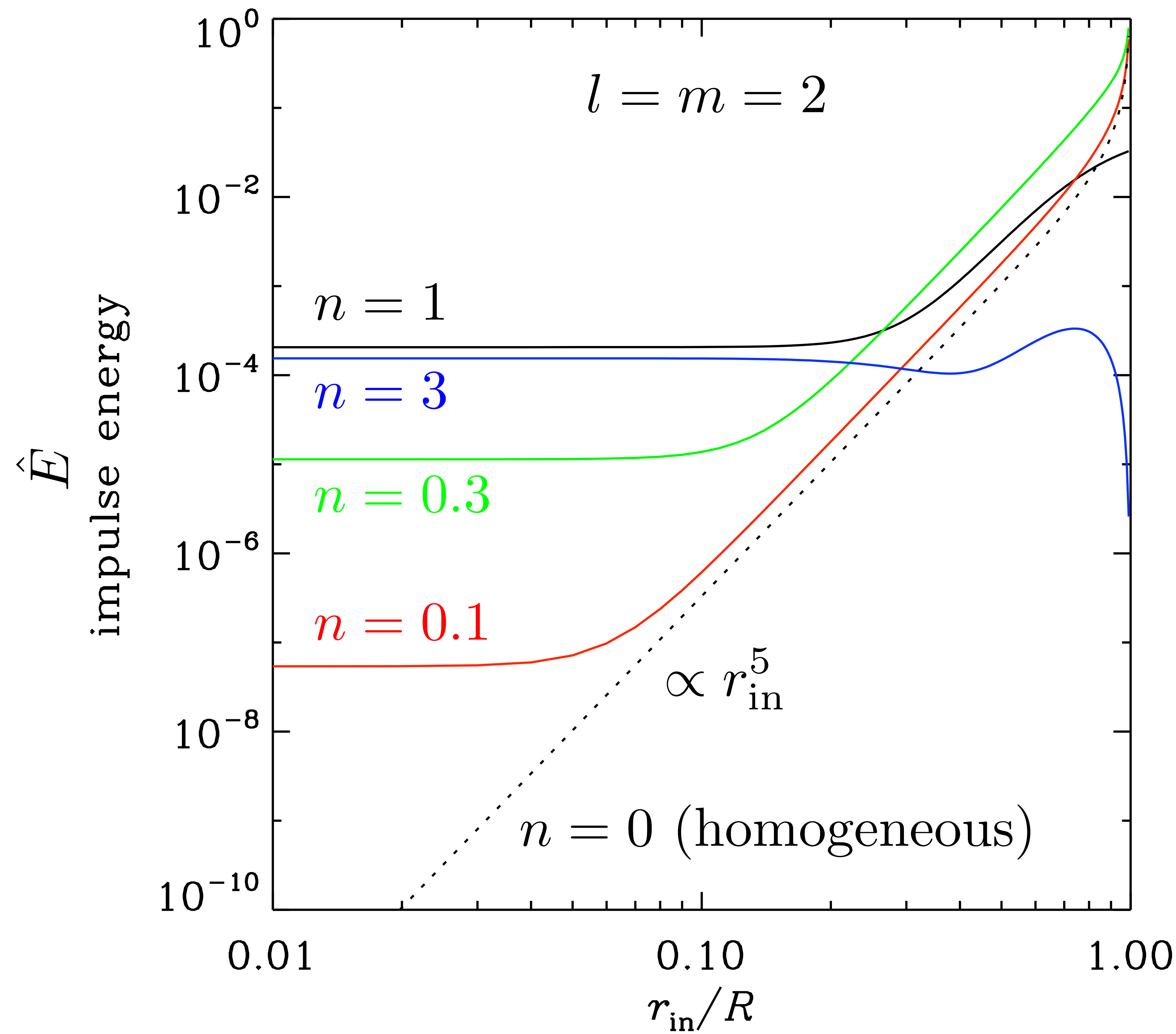
but beware trivial inertial modes with $l = 2$

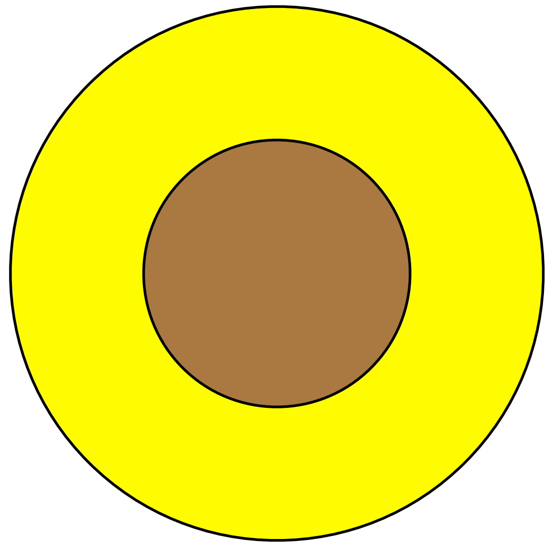
- Two homogeneous fluids
  - Similar but weaker result
  - Strengthened if densities differ greatly



Impulsive energy transfer / frequency-averaged dissipation

- Polytrope with rigid core $\quad p \propto \rho^{1+1/n}$

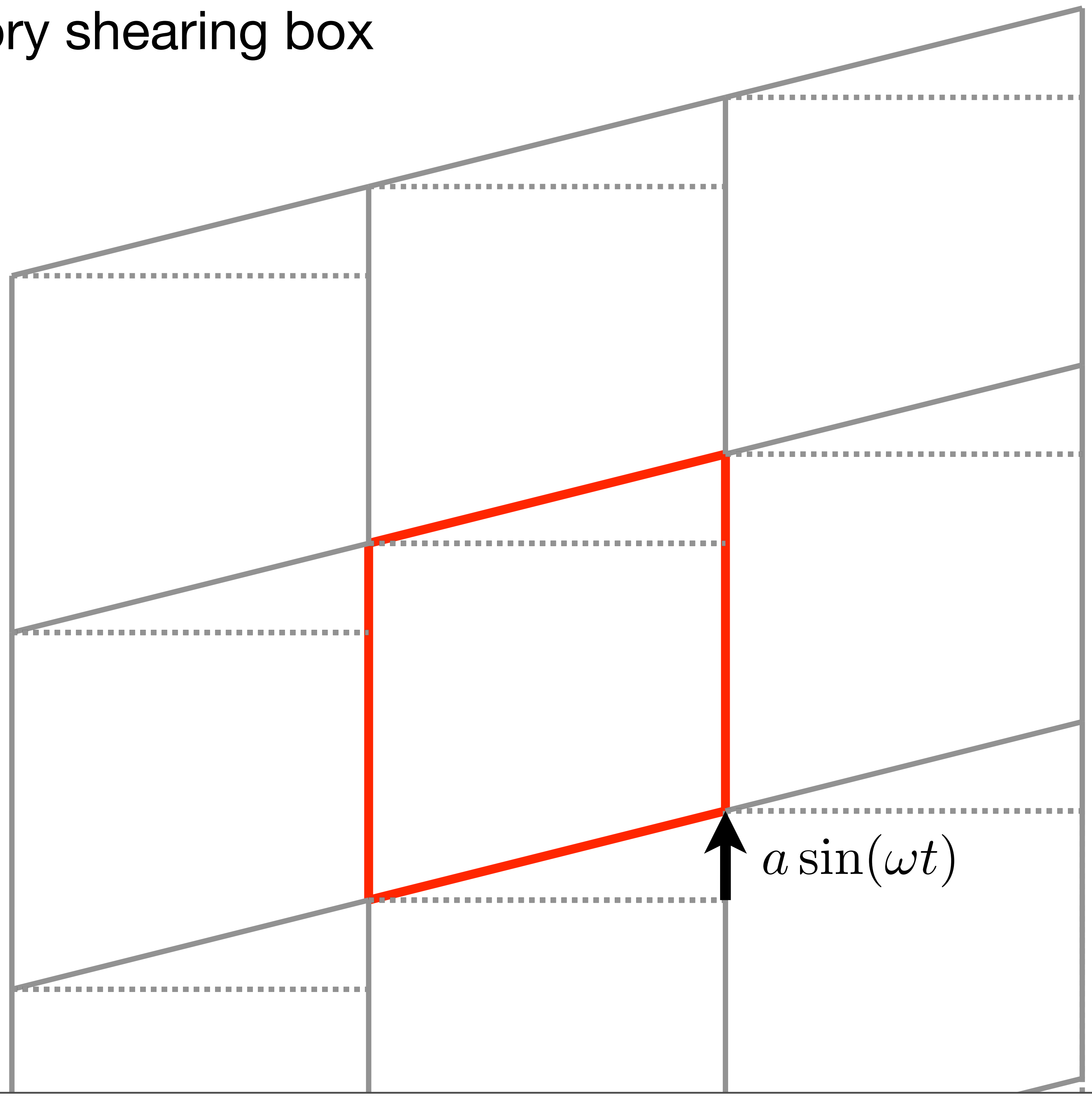




Monday, December 19, 2011

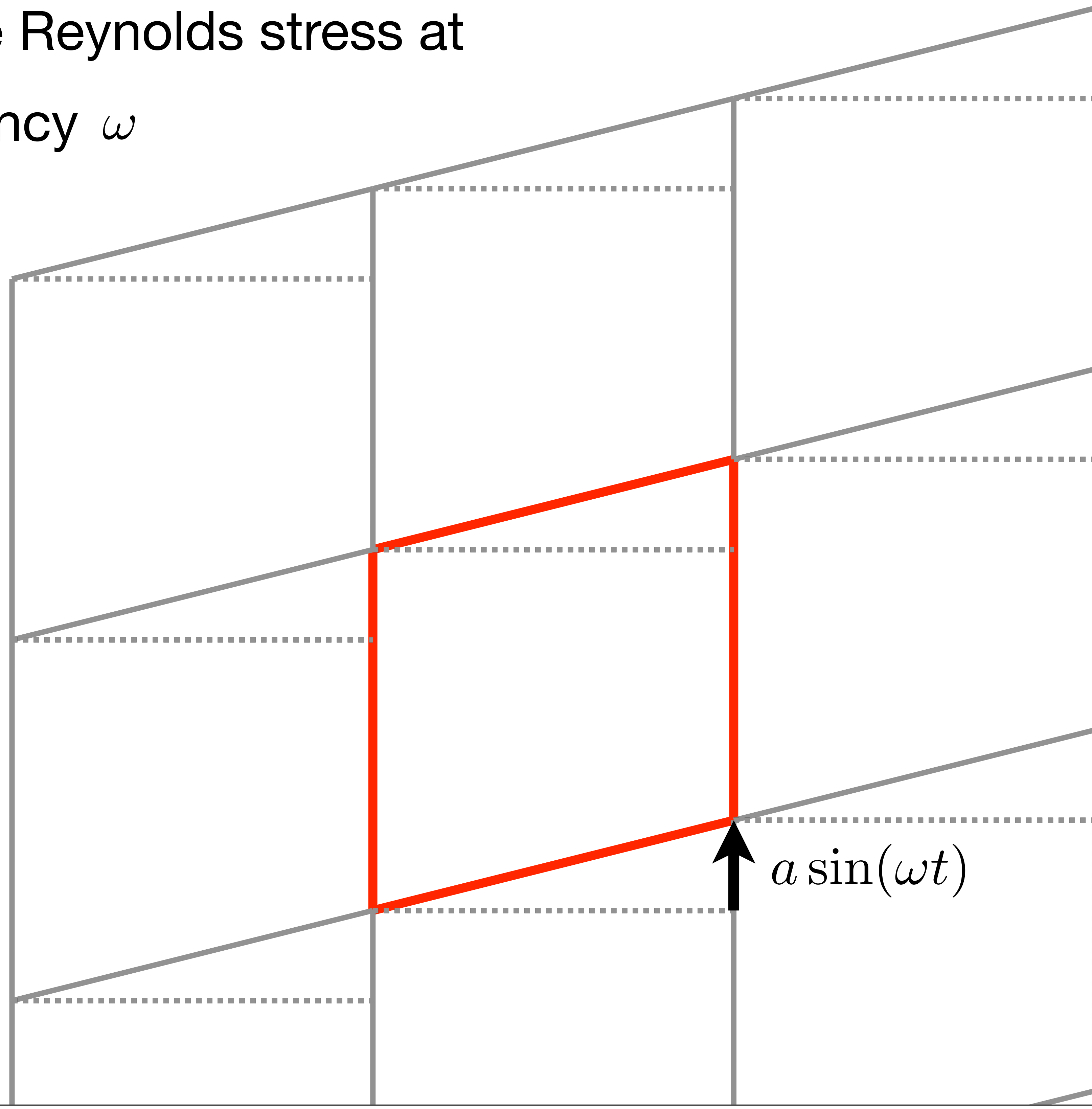
Effective viscosity of turbulent convection

- How does a convecting fluid respond to periodic distortion?
- Oscillatory shearing box



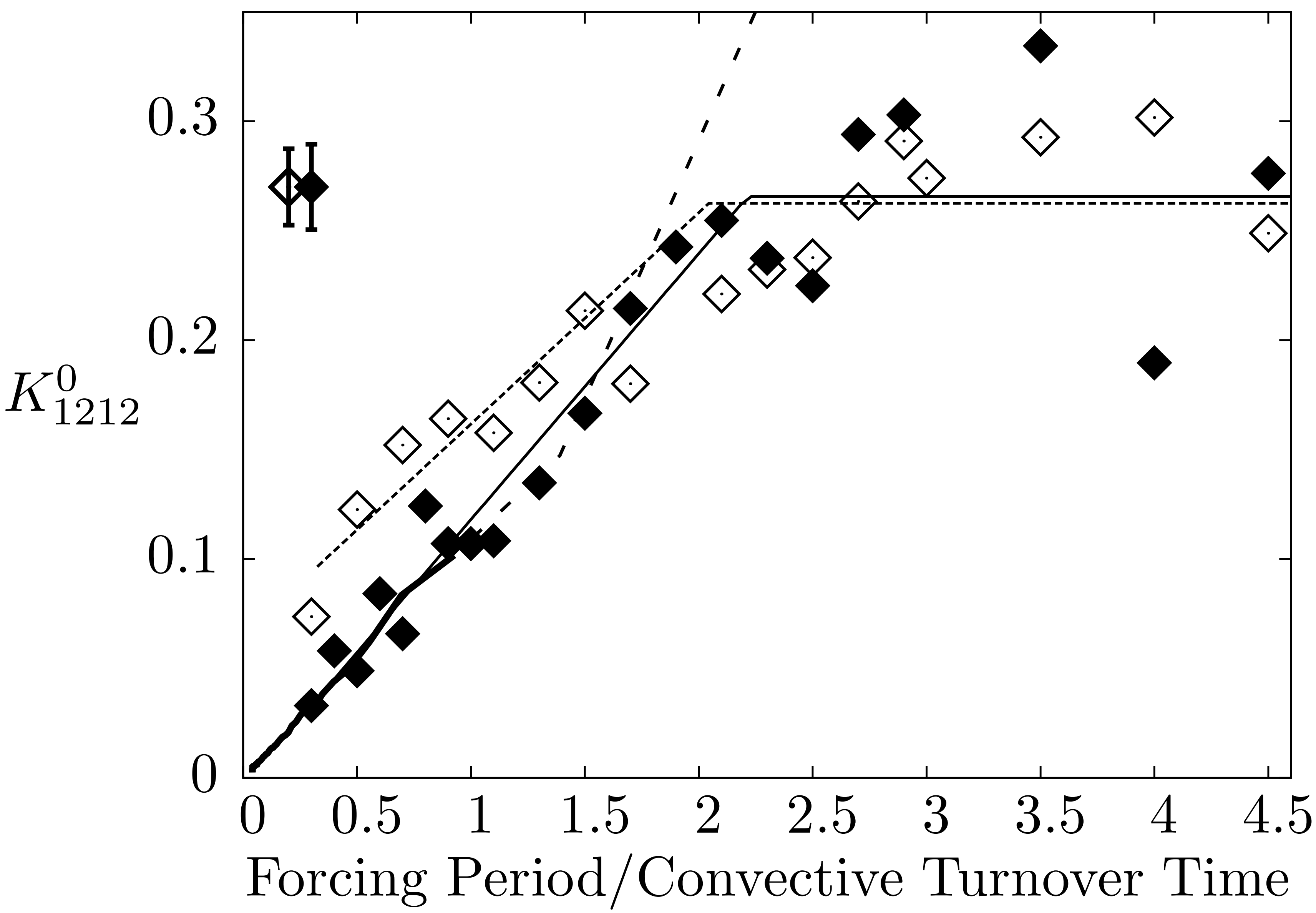
Effective viscosity of turbulent convection

- Compute convective or other flow in OSB
- Measure Reynolds stress at frequency ω

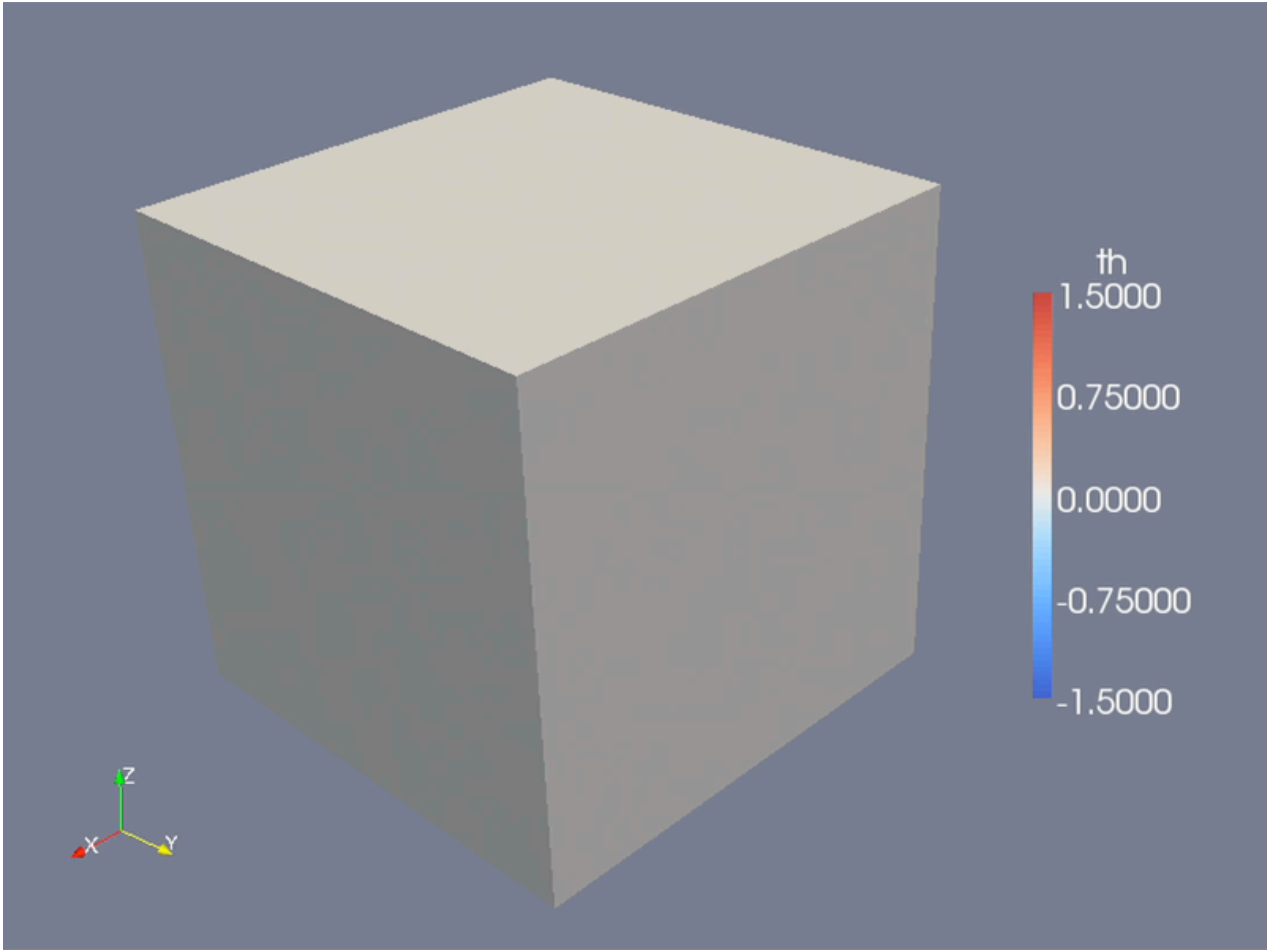


Previous hypotheses and results

- Zahn (1966) : viscosity $\propto \omega^{-1}$ (large eddies)
- Goldreich & Nicholson (1977) : viscosity $\propto \omega^{-2}$ (small eddies)
- Goodman & Oh (1997) : viscosity $\propto \omega^{-5/3}$ (small eddies)
- Penev et al. (2009) : viscosity $\propto \omega^{-1}$

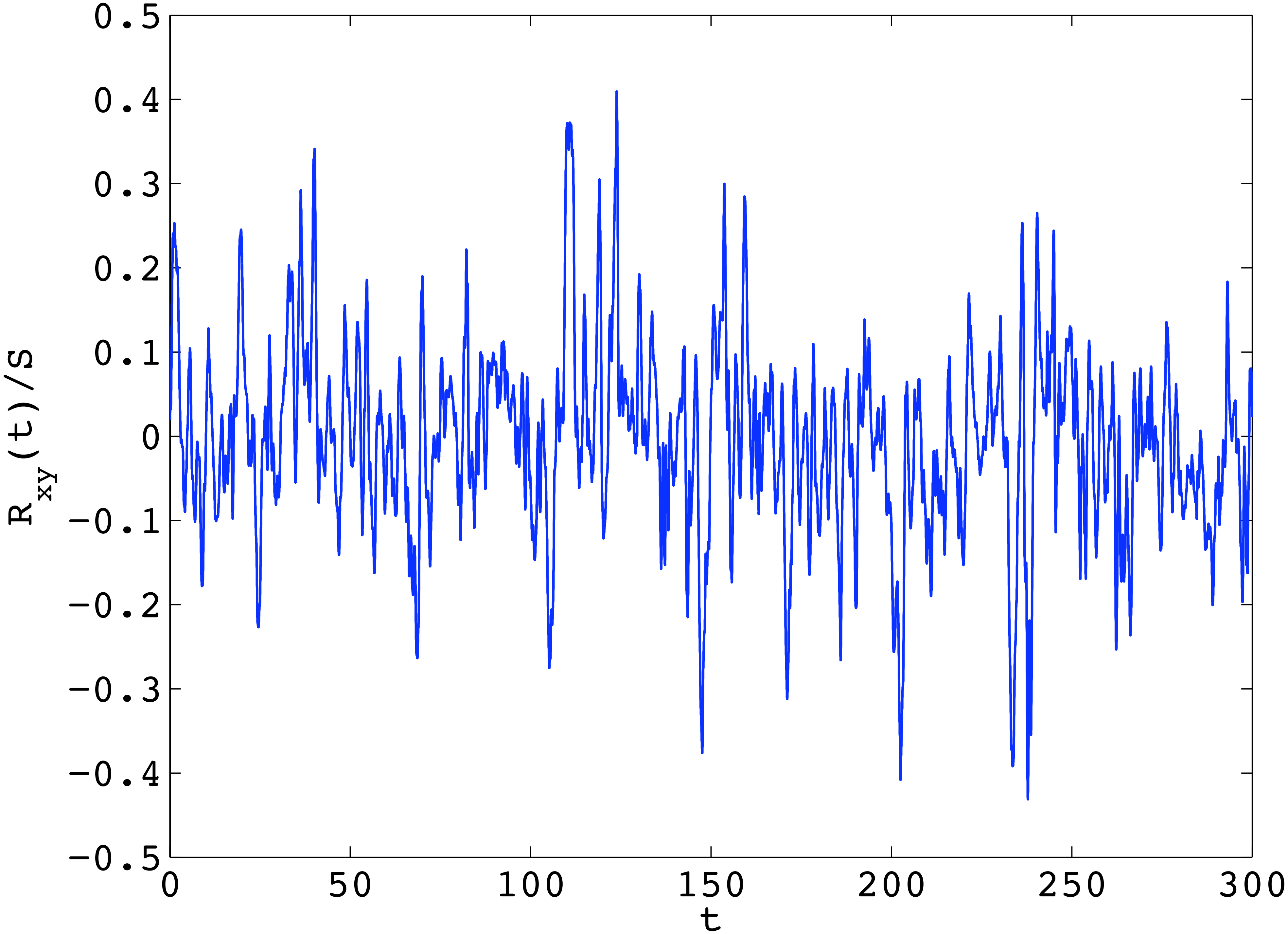


Convection in an oscillatory shearing box (Geoffroy Lesur)



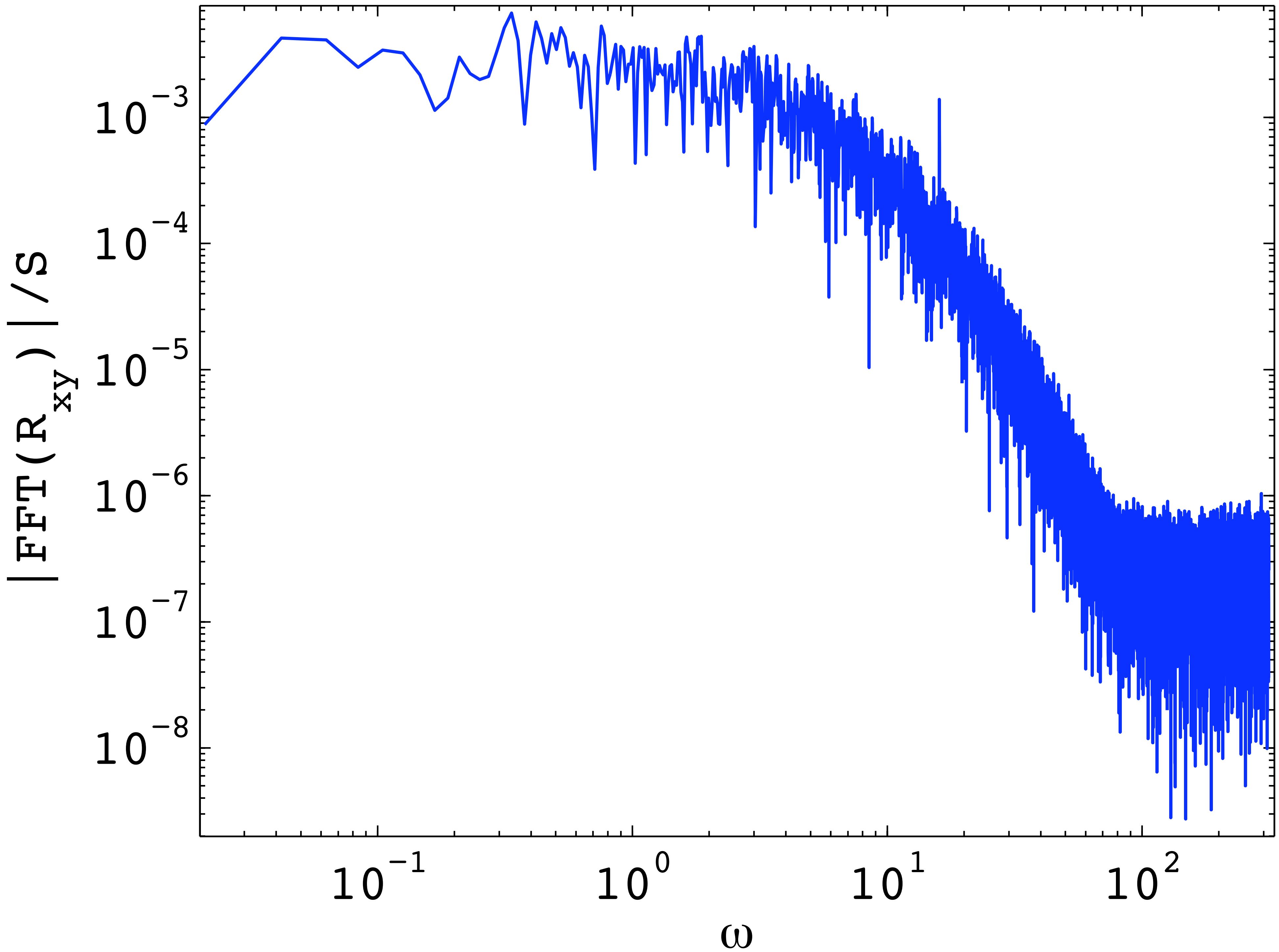
Convection in an oscillatory shearing box (Geoffroy Lesur)

- Time series of Reynolds stress (shear stress)



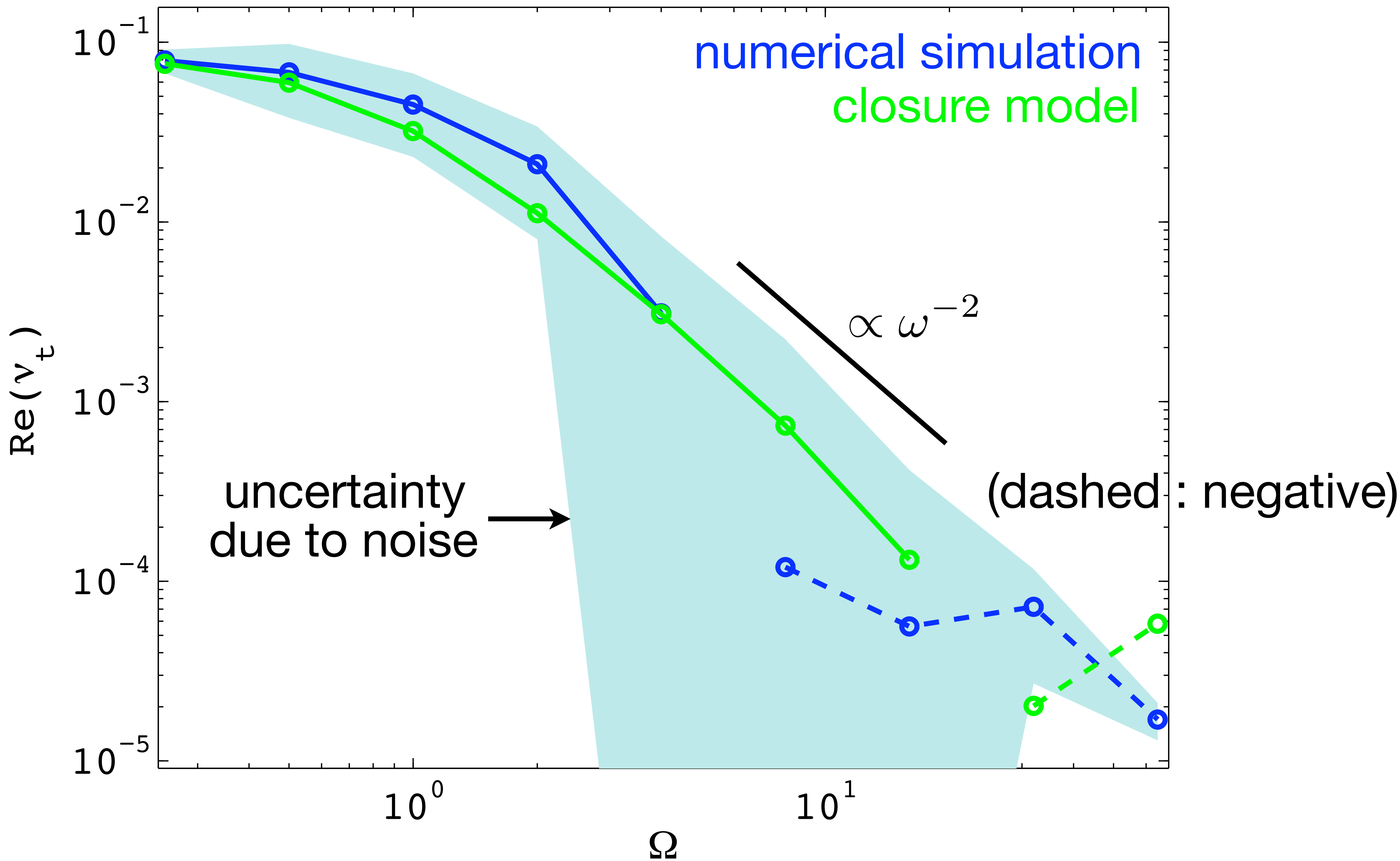
Convection in an oscillatory shearing box (Geoffroy Lesur)

- Fourier transform of Reynolds stress (shear stress)



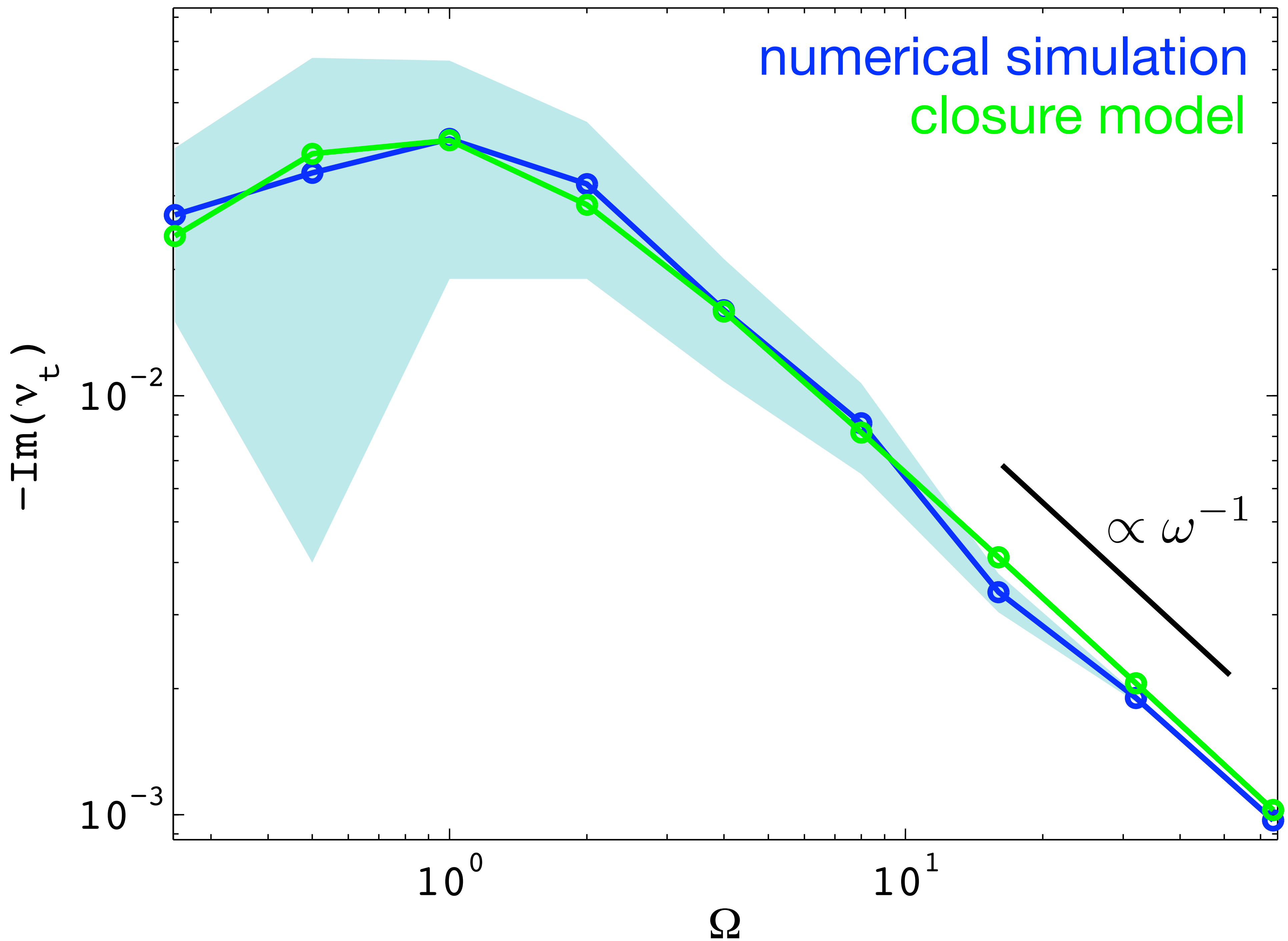
Convection in an oscillatory shearing box (Geoffroy Lesur)

- Real part of effective viscosity versus tidal frequency



Convection in an oscillatory shearing box (Geoffroy Lesur)

- Imaginary part of effective viscosity versus tidal frequency



Analytical results for high-frequency shear

General flow (laminar, turbulent, convective, ...)

Tidal period $\ll$ flow timescales (relevant for large eddies)

- Dominant response is elastic
- Next effect is viscosity $\propto \omega^{-2}$
- Coefficients may be positive, negative or zero depending on flow statistics, anisotropy, etc.
- Incompatible with Zahn (1966)
- Different from Goldreich & Nicholson (1977), Goodman & Oh (1997)
- Different from Penev et al. (2009)
- Raises the possibility of tidal anti-dissipation

Conclusions

- Idealized response (inertial waves) is highly frequency-dependent
- Frequency-averaged dissipation is robust and readily calculated
- For $l = m = 2$ dissipation is most efficient for :
  - larger, more rigid or denser cores
  - greater density stratification (larger polytropic index)
- Other (tesseral) harmonics excite richer response and may be important even though intrinsically weaker

- High-frequency tidal response of convection (and other flows) is :
  - elastic (+, – or 0)
  - viscous (+, – or 0), $\nu \propto \omega^{-2}$ and therefore small
  - anisotropic
- More work required for stellar or planetary application