Structural Evolution of Hot Jupiters Under Ohmic Dissipation

[1]Konstantin Batygin
[1]David J. Stevenson
[2]Peter Bodenheimer

[1]California Institute of Technology
[2]UC Santa Cruz

The Problem

Detection of first transiting planet, HD 209458b ($R = 1.35R_{Jup}$)

Standard gas-giant cooling theory suggests that evolved planets should not be significantly bigger than Jupiter.
(i.e. expect radius to shrink significantly in less than a Gyr)

Radius anomaly - HD 209458b is too puffy.

A decade of transit detections: puffy hot Jupiters are common!

What's going on?

Some Proposed Solutions

Inflation

I) Current eccentricity tides

(Bodenheimer et al 2001/2003, Mardling 2007, Liu et al 2008)

II) Strong early tidal heating

(Ibgui & Burrows 2009, Miller et al 2009, Laconte et al 2010)

III) Strong early tidal heating

(Arras & Socrates 2009, Gu & Oglivie 2009)

Forestalled Contraction

IV) Kinetic Heating

(Showman & Guillot 2002, Guillot & Showman 2002)

V) Enhanced opacity

(Burrows et al 2007)

VI) Double diffusive convection

(Stevenson 1985, Chabrier & Baraffe 2007)

etc...

Our Solution

Winds on hot Jupiters are fast (~ few km/s)

+

Atmospheres of hot Jupiters are hot (T > 1500K)

Thermal ionization, which results in electrical conductivity + fast wind allows for induction of emf

Electrical currents through the interior

Ohmic Dissipation in the atmosphere stalls secular cooling

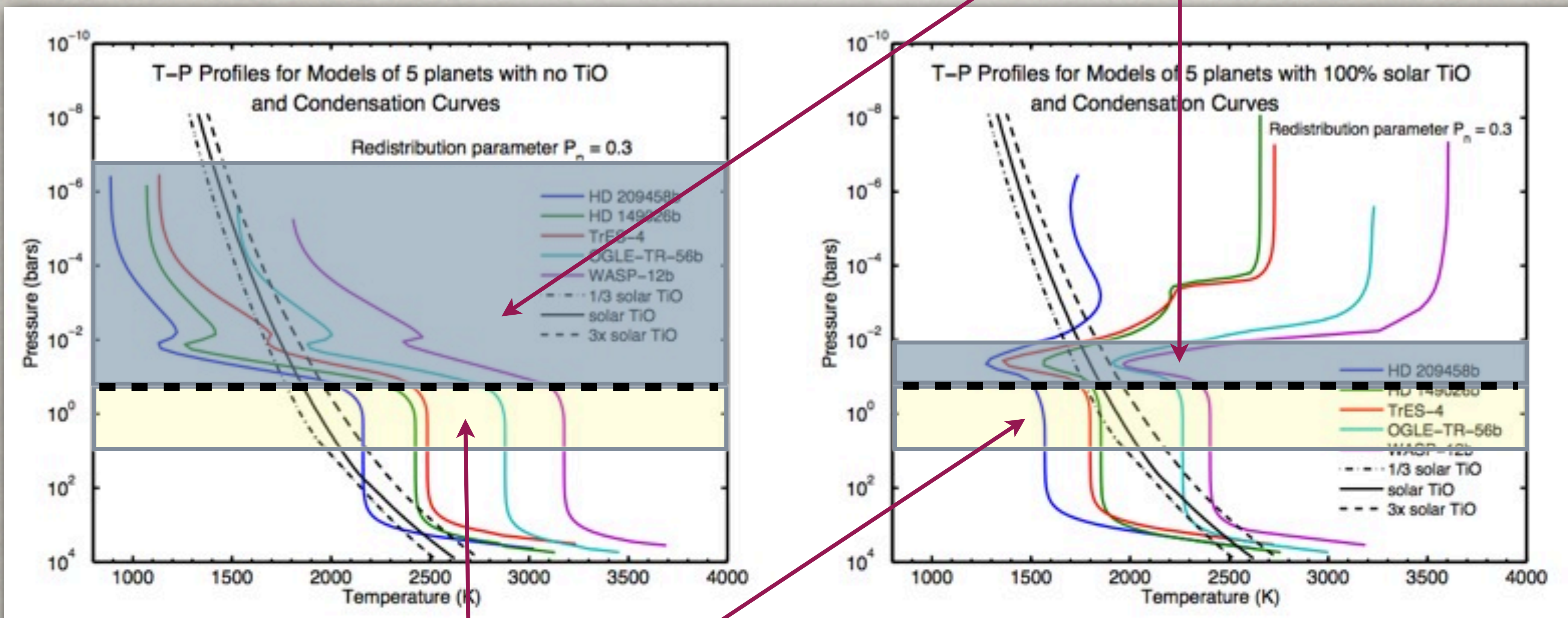
Ohmic Dissipation in the interior inflates the planet

Ionization in hot Jupiter atmospheres

$$\frac{n_j^+ n_e}{n_j - n_j^+} = \left(\frac{m_e k_b T}{2\pi\hbar^2}\right)^{\frac{3}{2}} \exp\left(-I_j/k_b T\right)$$

Zero Radial Current

Electrical Insulator



T-P profile from Spiegel et al 2009

Weather Layer (v x *B*)

Induced Current

$$\boxed{\frac{\partial \vec{B}}{\partial t} = -\vec{\nabla} \times \lambda \vec{\nabla} \times \vec{B} + \vec{\nabla} \times \left(\vec{v} \times \vec{B} \right)}$$

seek a steady-state solution

assume background field is a dipole field

assume the (v x B) term is dominated by the background field

Induced Current

$$\boxed{\frac{\partial \vec{B}}{\partial t} = -\vec{\nabla} \times \lambda \vec{\nabla} \times \vec{B} + \vec{\nabla} \times \left(\vec{v} \times \vec{B}\right)}$$

seek a steady-state solution

assume background field is a dipole field

assume the (v x B) term is dominated by the background field

$$\vec{\nabla} \times \lambda \left(\vec{\nabla} \times \vec{B}_{ind}\right) = \vec{\nabla} \times \left(\vec{v} \times \vec{B}_{dip}\right)$$

Induced Current

$$\boxed{\frac{\partial \vec{B}}{\partial t} = -\vec{\nabla} \times \lambda \vec{\nabla} \times \vec{B} + \vec{\nabla} \times \left(\vec{v} \times \vec{B} \right)}$$

seek a steady-state solution

assume background field is a dipole field

assume the (v x B) term is dominated by the background field

$$\vec{\nabla} \times \lambda \left(\vec{\nabla} \times \vec{B}_{ind} \right) = \vec{\nabla} \times \left(\vec{v} \times \vec{B}_{dip} \right)$$

electric field

$$\overrightarrow{J}_{ind} = \sigma \left(\overrightarrow{v} \times \overrightarrow{B}_{dip} - \vec{\nabla}\Phi \right)$$

Induced Current

$$\boxed{\frac{\partial \vec{B}}{\partial t} = -\vec{\nabla} \times \lambda \vec{\nabla} \times \vec{B} + \vec{\nabla} \times \left(\vec{v} \times \vec{B}\right)}$$

seek a steady-state solution

assume background field is a dipole field

assume the (v x B) term is dominated by the background field

$$\vec{\nabla} \times \lambda \left(\vec{\nabla} \times \vec{B}_{ind}\right) = \vec{\nabla} \times \left(\vec{v} \times \vec{B}_{dip}\right)$$

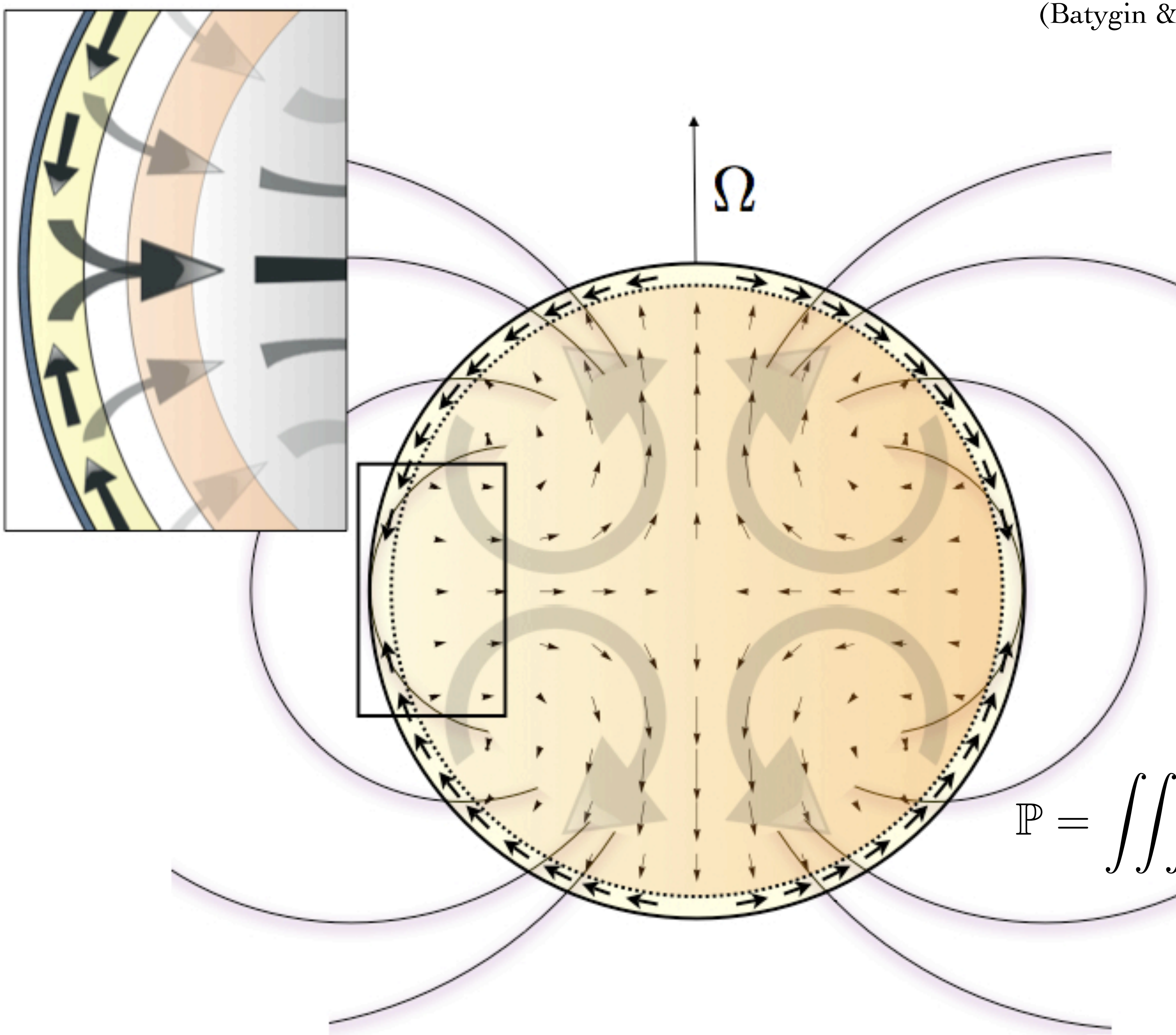
electric field

$$\overrightarrow{J}_{ind} = \sigma \left(\overrightarrow{v} \times \overrightarrow{B}_{dip} - \vec{\nabla}\Phi\right)$$

$\vec{v} \propto \sin(\theta)\ \hat{\phi}$

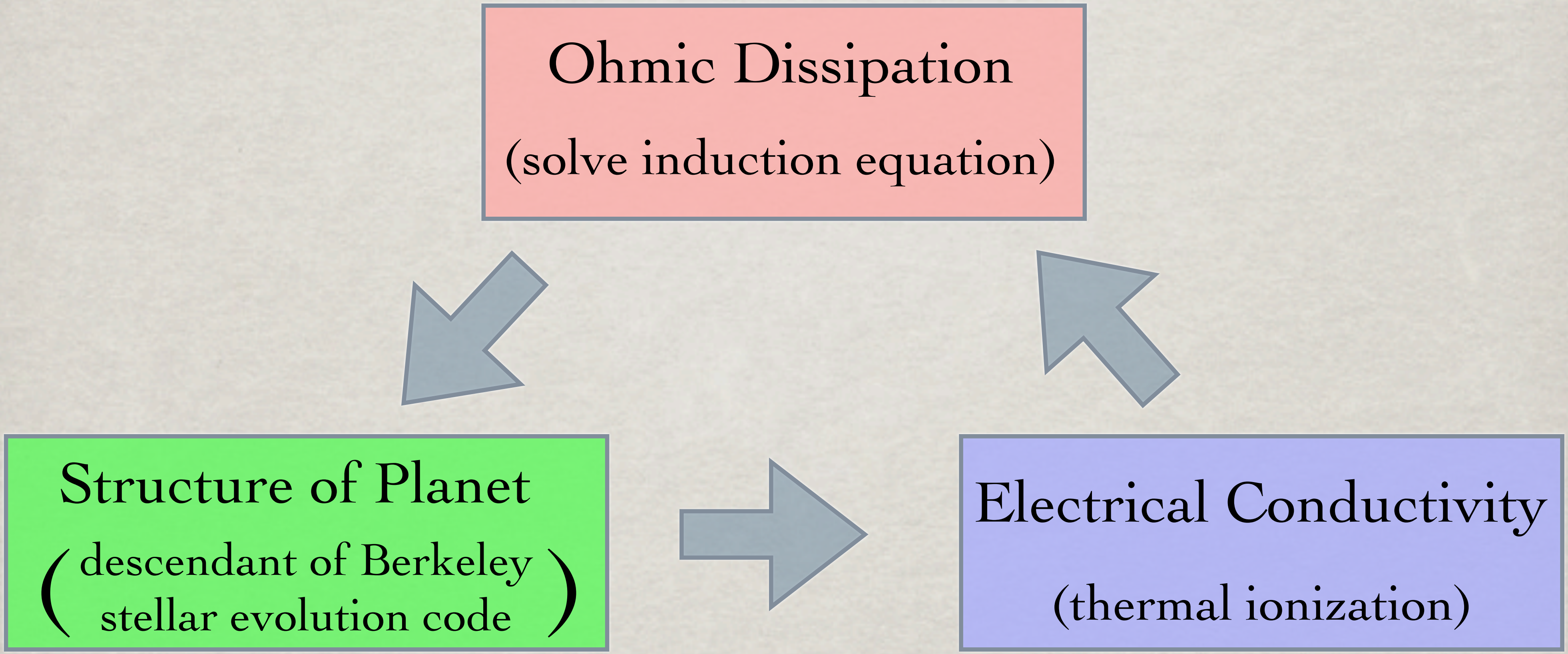
$$\boxed{\overrightarrow{\nabla} \cdot \sigma \overrightarrow{\nabla} \Phi = \overrightarrow{\nabla} \cdot \sigma \left(\overrightarrow{v} \times \overrightarrow{B}_{dip}\right)}$$

(Batygin & Stevenson 2010)



$$\mathbb{P} = \iiint \frac{\vec{J}^2}{\sigma(r)} dV$$

“Self-Consistent” Approach



Variables:

Planetary Mass, Effective Temperature

Global Energetics

Invicid Navier-Stokes: $$\frac{D\vec{v}}{Dt} = -2\vec{\Omega} \times \vec{v} - \frac{\vec{\nabla} P}{\rho} + \vec{g} + \frac{\vec{J} \times \vec{B}}{\rho}$$

Global Energetics

Invicid Navier-Stokes:

$$\frac{D\vec{v}}{Dt} = -2\vec{\Omega} \times \vec{v} - \frac{\vec{\nabla} P}{\rho} + \vec{g} + \frac{\vec{J} \times \vec{B}}{\rho}$$

Change in energy due to the Lorentz Force:

$$\left(\frac{\rho}{2} \frac{Dv^2}{Dt} \right)_L = \vec{v} \cdot \vec{J} \times \vec{B} = -\vec{J} \cdot \vec{v} \times \vec{B}$$

Global Energetics

Invicid Navier-Stokes: $$\frac{D\vec{v}}{Dt} = -2\vec{\Omega} \times \vec{v} - \frac{\vec{\nabla} P}{\rho} + \vec{g} + \frac{\vec{J} \times \vec{B}}{\rho}$$

Change in energy due to the Lorentz Force: $$\left(\frac{\rho}{2} \frac{Dv^2}{Dt} \right)_L = \vec{v} \cdot \vec{J} \times \vec{B} = -\vec{J} \cdot \vec{v} \times \vec{B}$$

Ohmic dissipation

???

By Ohm's Law: $$-\vec{J} \cdot \vec{v} \times \vec{B} = \boxed{-\frac{J^2}{\sigma}} - \vec{J} \cdot \nabla \Phi$$

Global Energetics

Invicid Navier-Stokes:

$$\frac{D\vec{v}}{Dt} = -2\vec{\Omega} \times \vec{v} - \frac{\vec{\nabla} P}{\rho} + \vec{g} + \frac{\vec{J} \times \vec{B}}{\rho}$$

Change in energy due to the Lorentz Force:

$$\left(\frac{\rho}{2}\frac{Dv^2}{Dt}\right)_L = \vec{v} \cdot \vec{J} \times \vec{B} = -\vec{J} \cdot \vec{v} \times \vec{B}$$

Ohmic dissipation

???

By Ohm's Law:

$$-\vec{J} \cdot \vec{v} \times \vec{B} = \boxed{-\frac{J^2}{\sigma}} - \vec{J} \cdot \nabla\Phi$$

Kill the second term on the RHS:

$$\int\int\int \vec{J} \cdot \nabla\Phi dV = \int\int\int \nabla \cdot (\vec{J}\Phi) dV = \oint (\vec{J}\Phi) \cdot d\vec{a} = 0$$

Global Energetics

Invicid Navier-Stokes: $$\frac{D\vec{v}}{Dt} = -2\vec{\Omega}\times\vec{v} - \frac{\vec{\nabla}P}{\rho} + \vec{g} + \frac{\vec{J}\times\vec{B}}{\rho}$$

Change in energy due to the Lorentz Force: $$\left(\frac{\rho}{2}\frac{Dv^2}{Dt}\right)_L = \vec{v}\cdot\vec{J}\times\vec{B} = -\vec{J}\cdot\vec{v}\times\vec{B}$$

Ohmic dissipation

???

By Ohm's Law: $$-\vec{J}\cdot\vec{v}\times\vec{B} = \boxed{-\frac{J^2}{\sigma}} - \vec{J}\cdot\nabla\Phi$$

Kill the second term on the RHS: $$\int\int\int \vec{J}\cdot\nabla\Phi dV = \int\int\int \nabla\cdot(\vec{J}\Phi)dV = \oint(\vec{J}\Phi)\cdot d\vec{a} = 0$$

$$\boxed{\int\int\int\left(\frac{\rho}{2}\frac{Dv^2}{Dt}\right)dV = -\int\int\int\frac{J^2}{\sigma}dV}$$

<u>Ohmic dissipation is work done by the flow!</u>

Mechanism Efficiency

In steady state, work done by the flow is limited by the efficiency factor i.e. the fraction of insolation that is available to do useful work

$$-\int\int\int \frac{J^2}{\sigma} dV = \boxed{\epsilon}\, \sigma_{\rm sf} T_{\rm eff}^4 \, \pi R^2$$

So what's the value of the efficiency factor?
In detail, a complex issue, that requires numerical MHD, but

$$\frac{D\vec{v}}{Dt} = ... + \frac{\vec{J}\times\vec{B}}{\rho} \sim ... + \frac{\sigma\vec{v}\vec{B}^2}{\rho} \sim ... - \frac{\vec{v}}{\tau_L}$$

$$\Downarrow$$

Massage the equations until they resemble Ekman balance

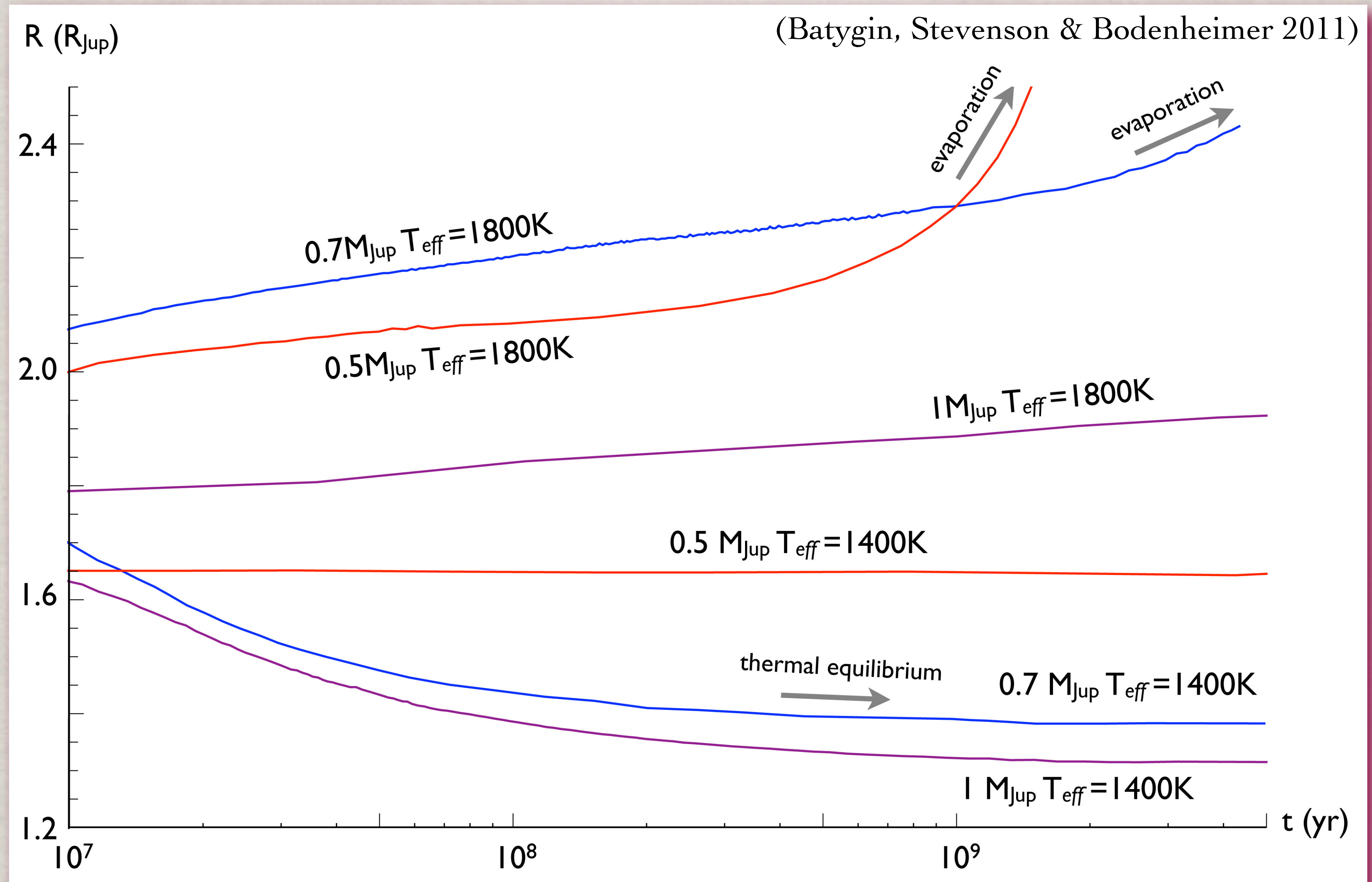
After some calculus + algebra, it can be shown that

$$\underbrace{\boxed{||\vec{v}|| \sim \frac{gH}{fR}\frac{\Delta T}{\langle T\rangle}}}_{\text{Thermal wind}} \left(1 + \underbrace{\boxed{\left(\frac{1}{f\tau_L}\right)^2}}_{\text{Magnetic drag}} + \underbrace{\boxed{\left(\frac{\overbrace{N}^{\text{Brunt-Vaisala}}}{f}\right)^2 \left(\frac{H}{R}\right)^2 \left(\frac{\overbrace{\tau_N}^{\text{Newtonian cooling timescale}}}{\tau_L}\right)}}_{\text{Reduction in eq-pole T-grad.}}\right)^{-1}$$

In the hot-Jupiter parameter regime, this gives

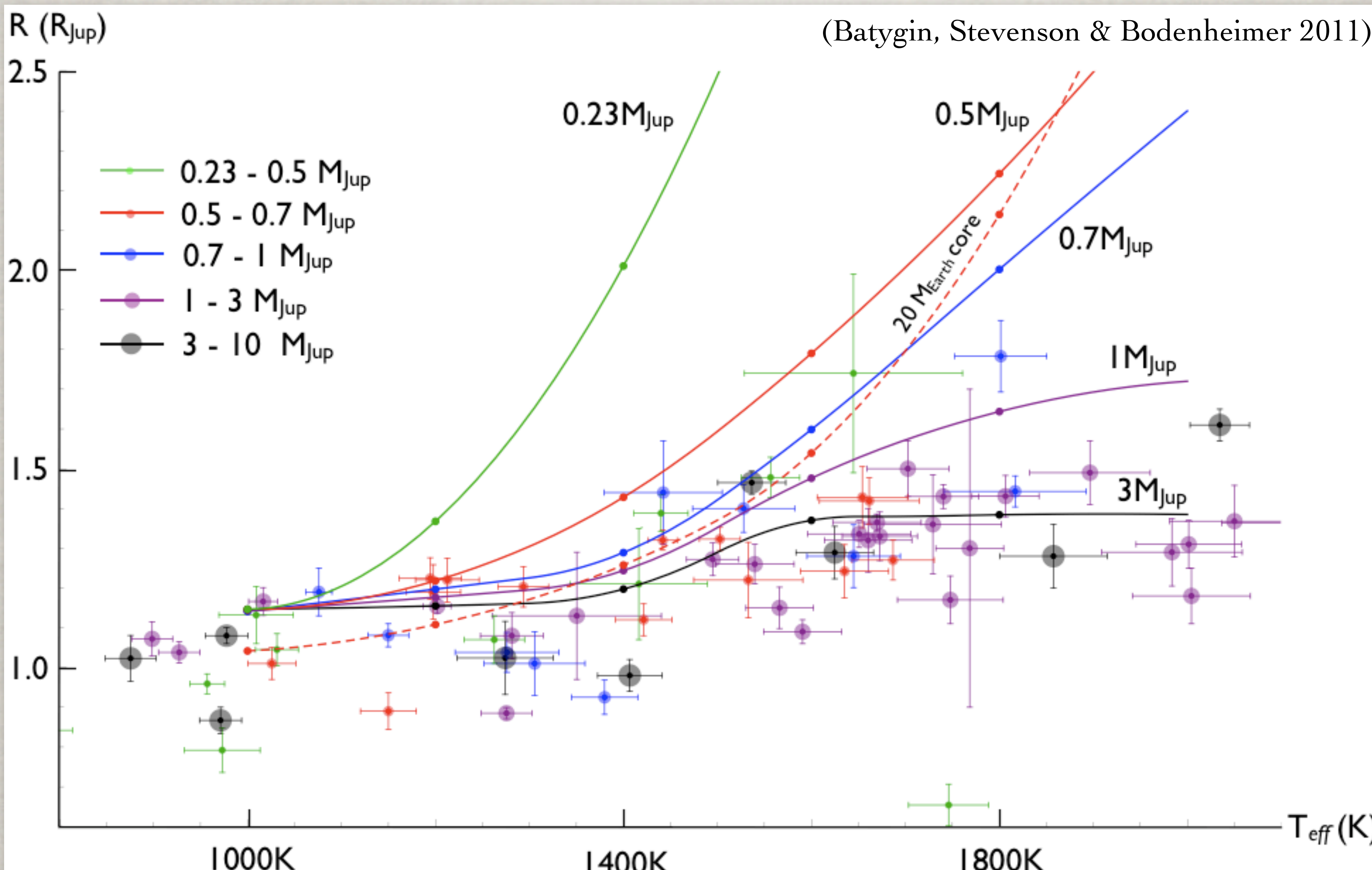
$$\epsilon = \frac{\mathbb{P}}{\sigma_{sf}T^4} = \frac{\rho H (\vec{v})^2}{\tau_L \sigma_{sf} T^4} \sim 3\% \quad \text{or a bit more.}$$

Thermal Evolution of Some Planets (ε = 3%)

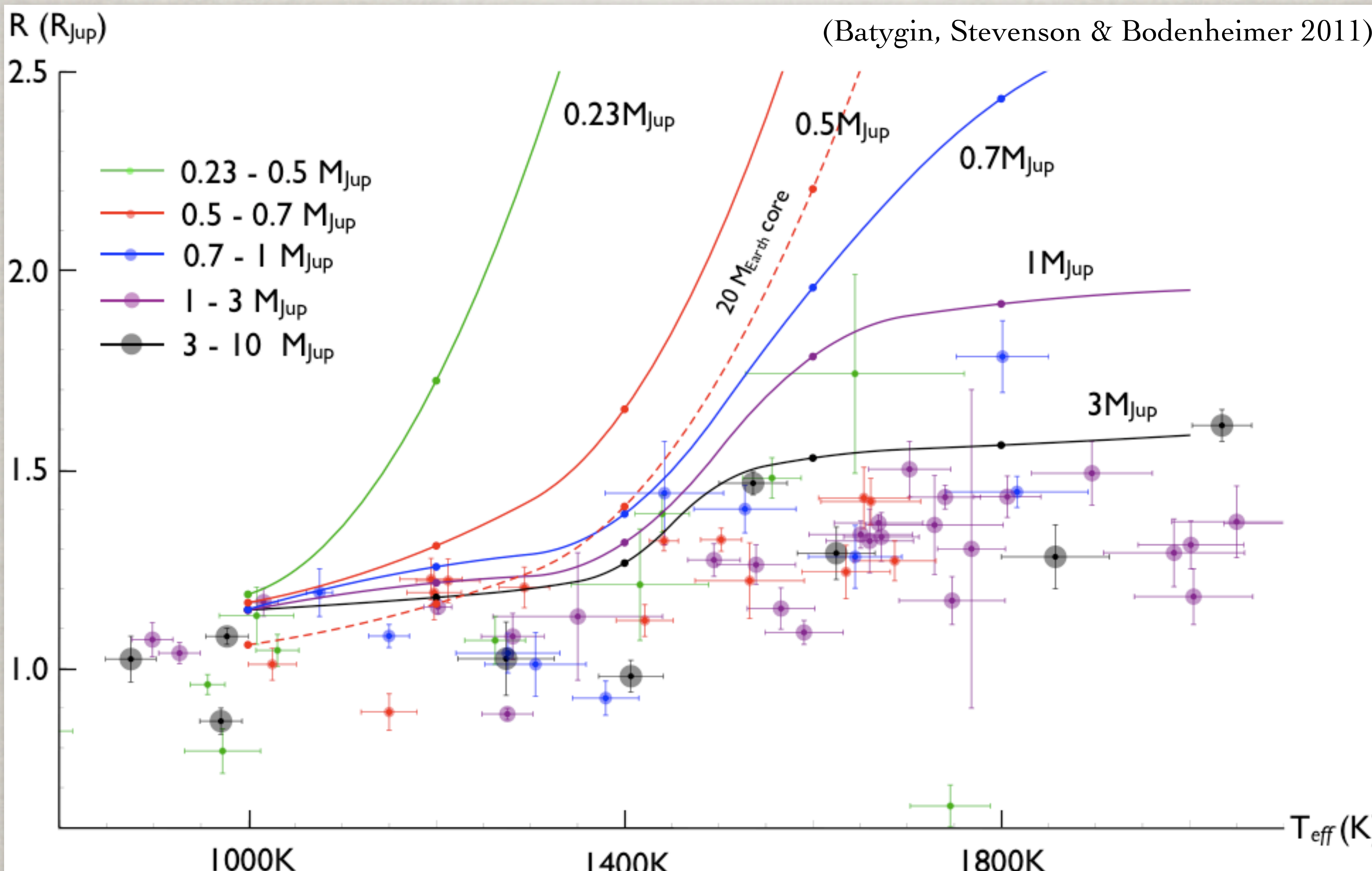


Hottest planets Ohmically heat up faster than they cool.

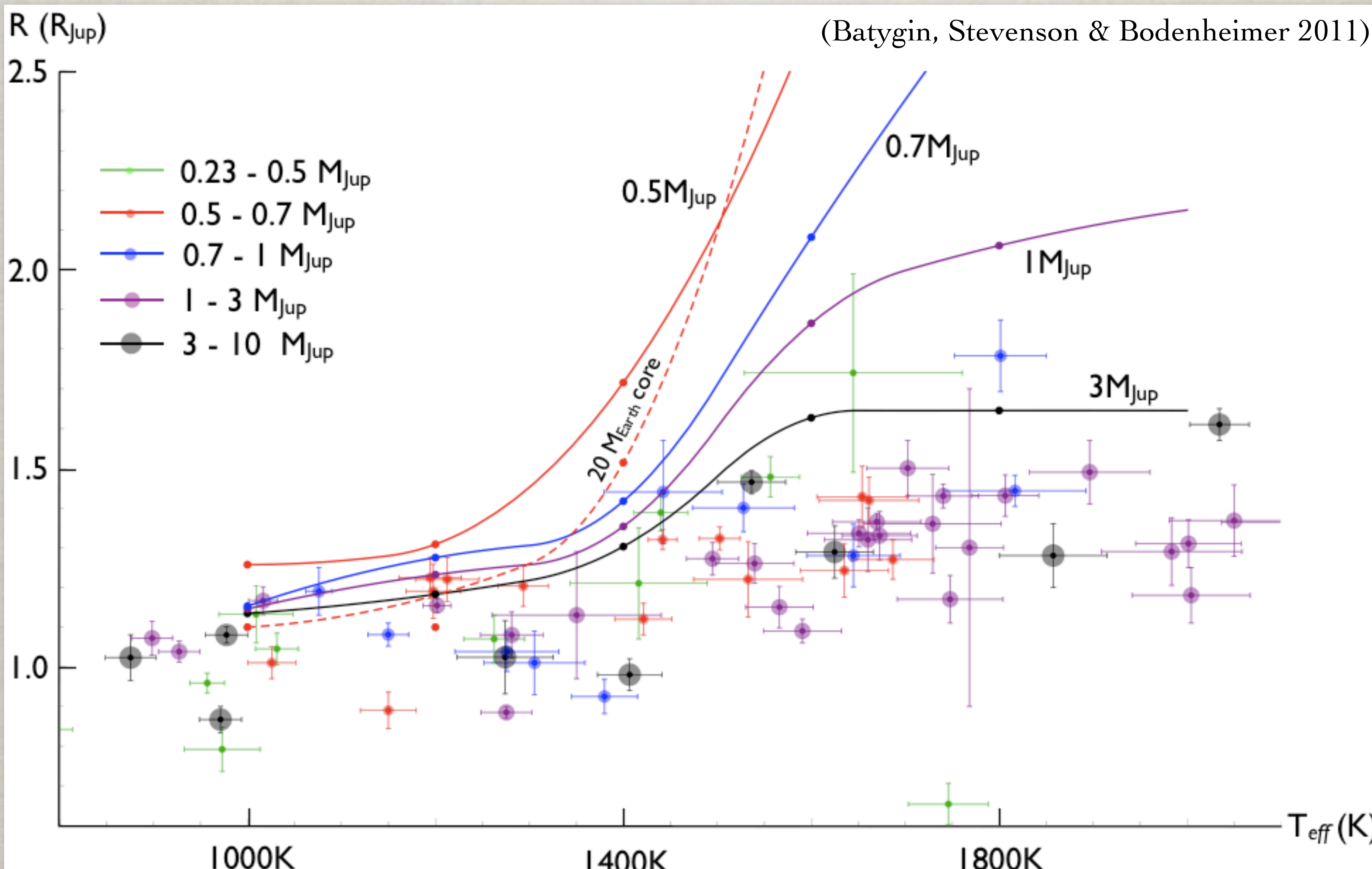
Theory (ε = 1%, t = 5Gyr) vs Data



Theory (ε = 3%, t = 5*Gyr*) vs Data



Theory (ε = 5%, t = 5Gyr) vs Data



Summary

Batygin & Stevenson 2010, ApJL 714
Batygin, Stevenson & Bodenheimer, 2011, ApJ 738

A new MHD mechanism for inflation of extrasolar gas giants.

Coupled structural/heating calculations show that the mechanism is universally capable of explaining radius anomalies

Radius is a strong function of mass, T_{eff}

Roche-lobe overflow is possible for low-mass hot Jupiters in the absence of high-Z cores

Future Work

We considered kinematic flows, but Lorentz force (JxB) may act to significantly modify the nature of the flow.

Reexamination of the results with a better atmospheric model, taking into account variability in the efficiency factor.

How does the induced current affect the interior dynamo?

What about the stellar magnetic field? Linking of field lines?

10^5

10^3

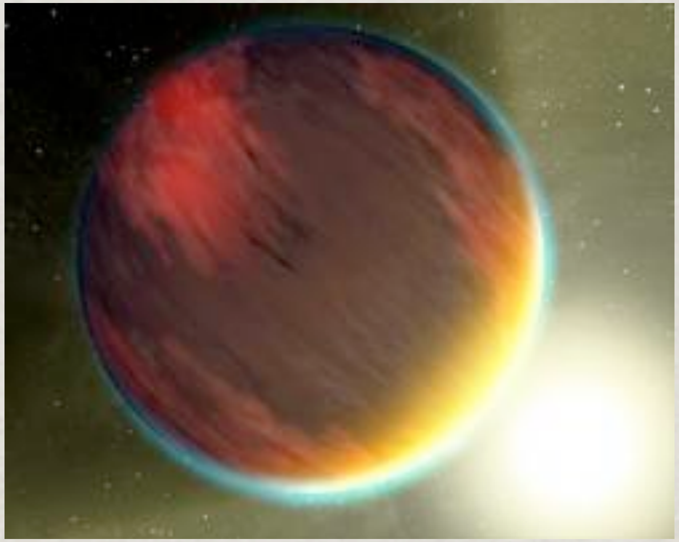
$$Re_m = \frac{VL}{\lambda} \gtrsim 1$$

$$\frac{1}{\mu_0 \sigma}$$

10^{-6}

only need 10^{-2} S/m !

Thank You



An extended thanks to my partners in crime

Dave Stevenson
Peter Bodenheimer
Mike Brown
Greg Laughlin
Alessandro Morbidelli
Sabine Stanley
Kleomenis Tsiganis

Some Scalings

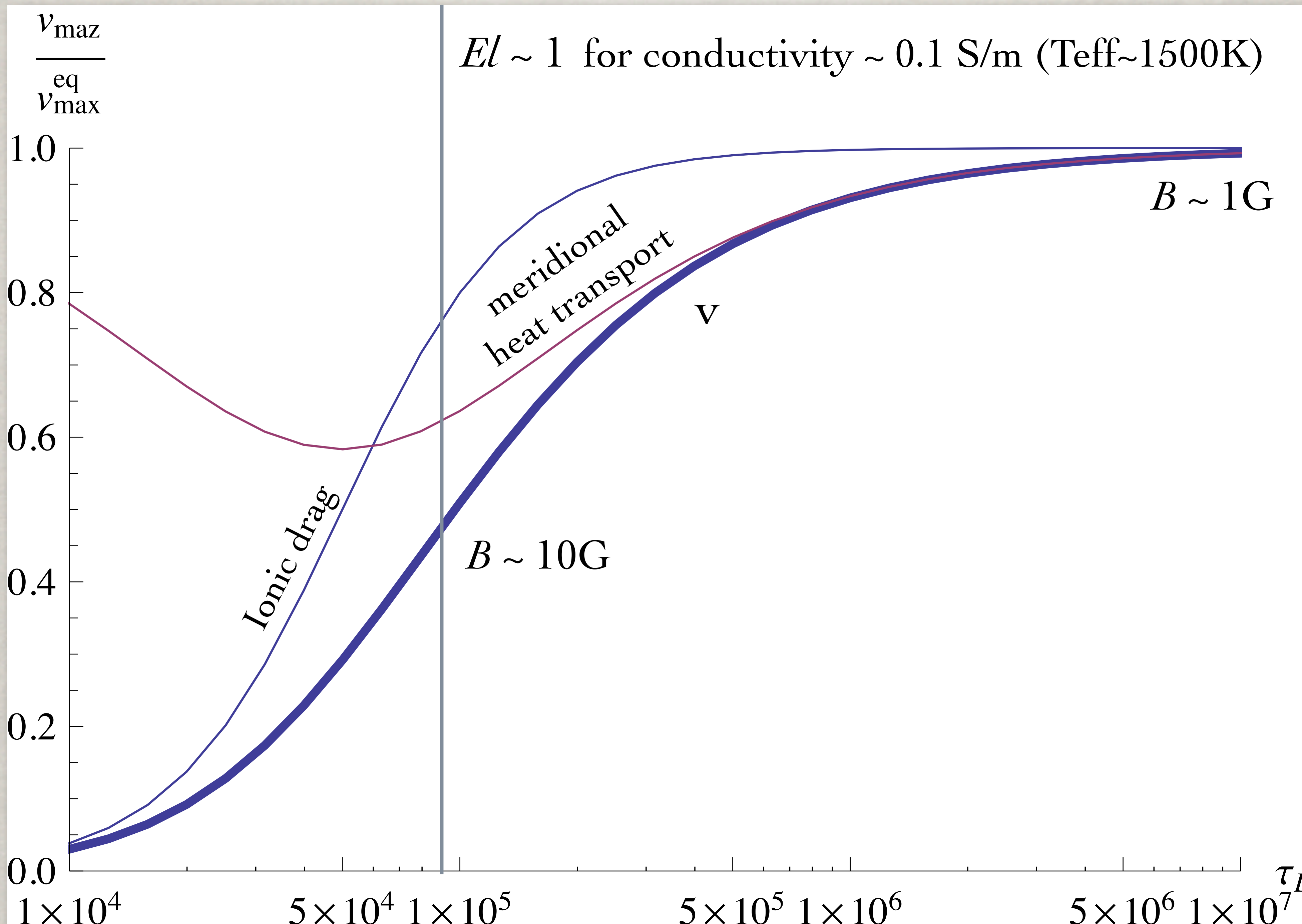
$$\left.\begin{array}{l} \mathbb{P} \propto \sqrt{Z} \\ \boxed{\mathbb{P} \propto \exp(T)} \end{array}\right\} \quad \mathbb{P} \propto \sigma$$

Changing Y (core vs. no core) has little effect.

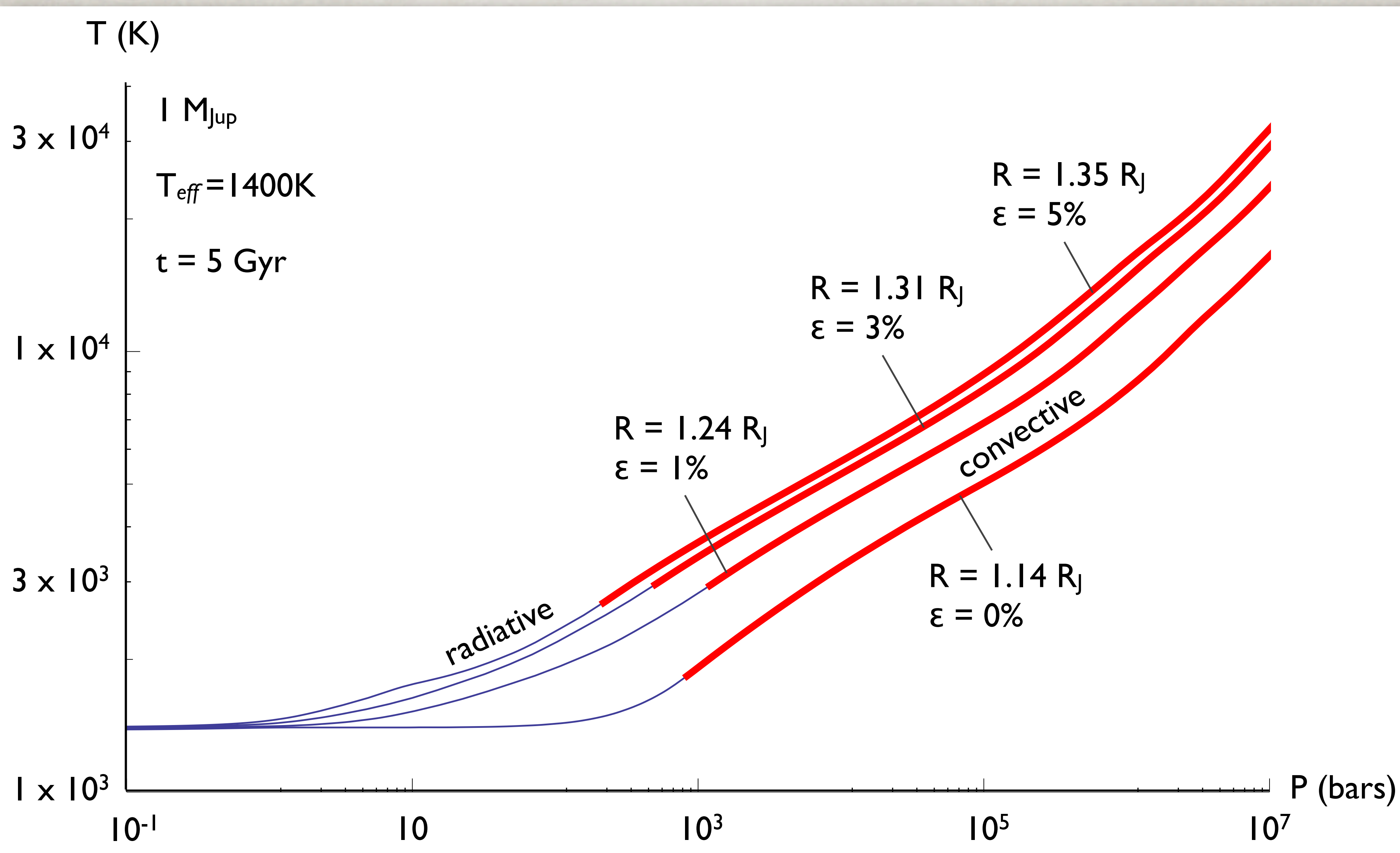
Also, <u>to leading order</u>,

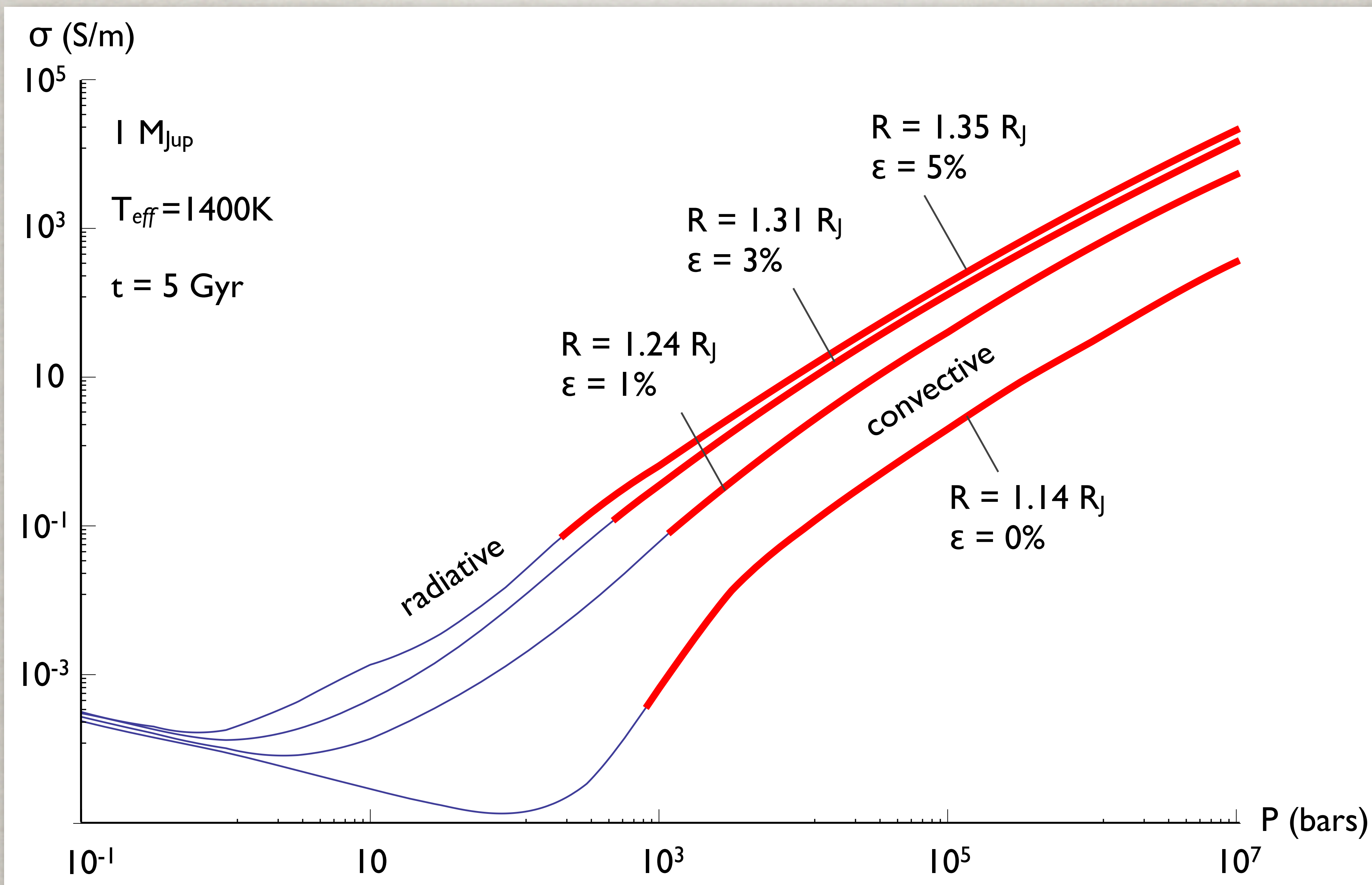
$$\mathbb{P}_{atm} \propto \delta \qquad \mathbb{P}_{int} \propto \delta^2$$

$$\boxed{\mathbb{P} \propto v^2} \qquad \boxed{\mathbb{P} \propto B^2}$$

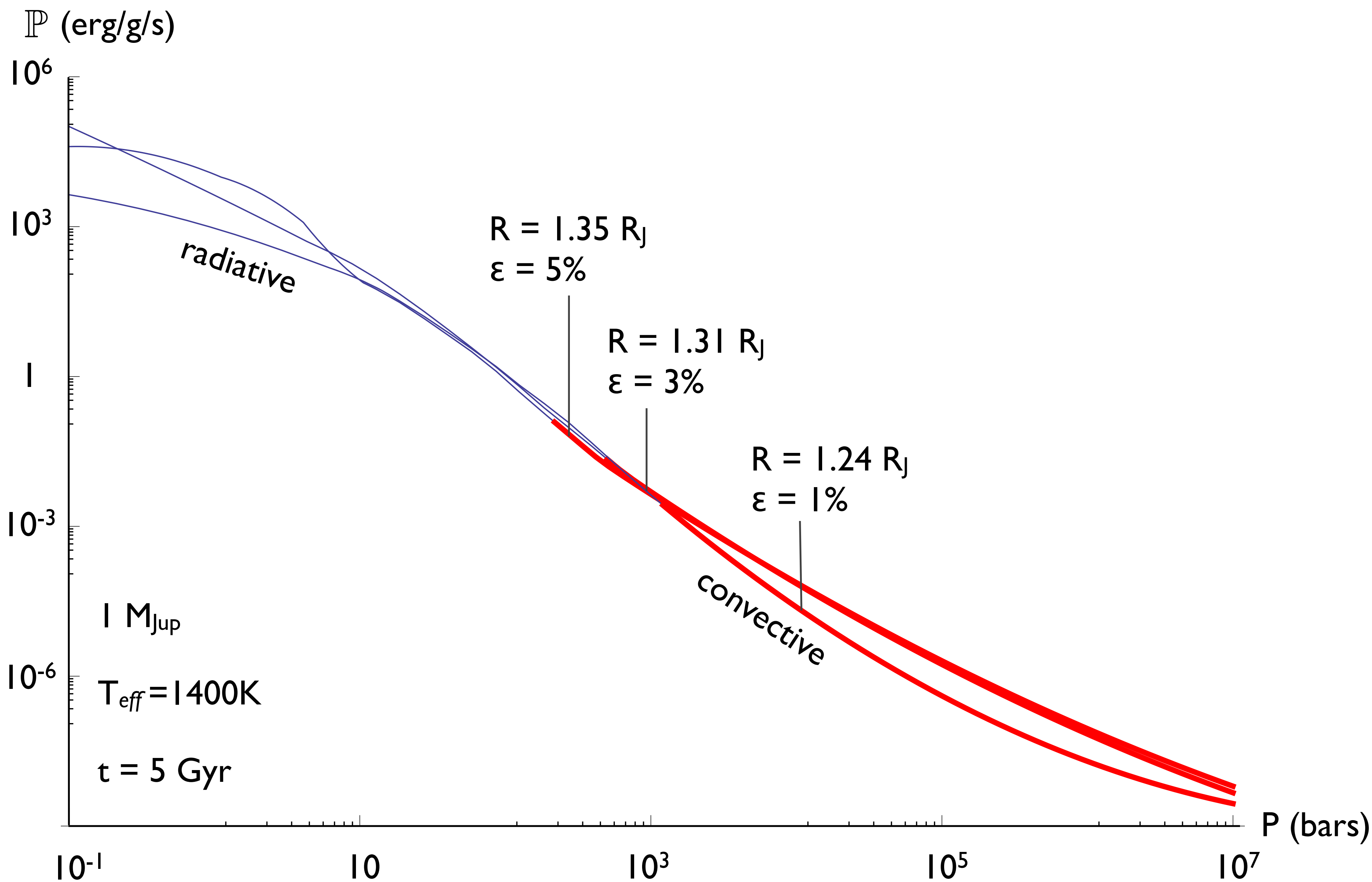


$\frac{v_{maz}}{v^{eq}_{max}}$
$El \sim 1$ for conductivity ~ 0.1 S/m (Teff~1500K)
1.0
0.8
0.6
0.4
0.2
0.0
meridional heat transport
v
Ionic drag
$B \sim 10$G
$B \sim 1$G
1×10^4
5×10^4
1×10^5
5×10^5
1×10^6
5×10^6
1×10^7
τ_L


T (K)
1 MJup
Teff=1400K
t = 5 Gyr
3 x 10^4
1 x 10^4
3 x 10^3
1 x 10^3
R = 1.35 RJ
ε = 5%
R = 1.31 RJ
ε = 3%
R = 1.24 RJ
ε = 1%
R = 1.14 RJ
ε = 0%
radiative
convective
P (bars)
10^-1
10
10^3
10^5
10^7



σ (S/m)
10^5
10^3
10
10^-1
10^-3
1 M_Jup
T_eff=1400K
t = 5 Gyr
R = 1.35 R_J
ε = 5%
R = 1.31 R_J
ε = 3%
R = 1.24 R_J
ε = 1%
R = 1.14 R_J
ε = 0%
radiative
convective
P (bars)
10^-1
10
10^3
10^5
10^7


ℙ (erg/g/s)
10^6
10^3
1
10^-3
10^-6
radiative
convective
R = 1.35 R_J
ε = 5%
R = 1.31 R_J
ε = 3%
R = 1.24 R_J
ε = 1%
1 M_Jup
T_eff =1400K
t = 5 Gyr
P (bars)
10^-1
10
10^3
10^5
10^7