Gaussian processes: the next step in exoplanet data analysis

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... let the data speak

University of Oxford

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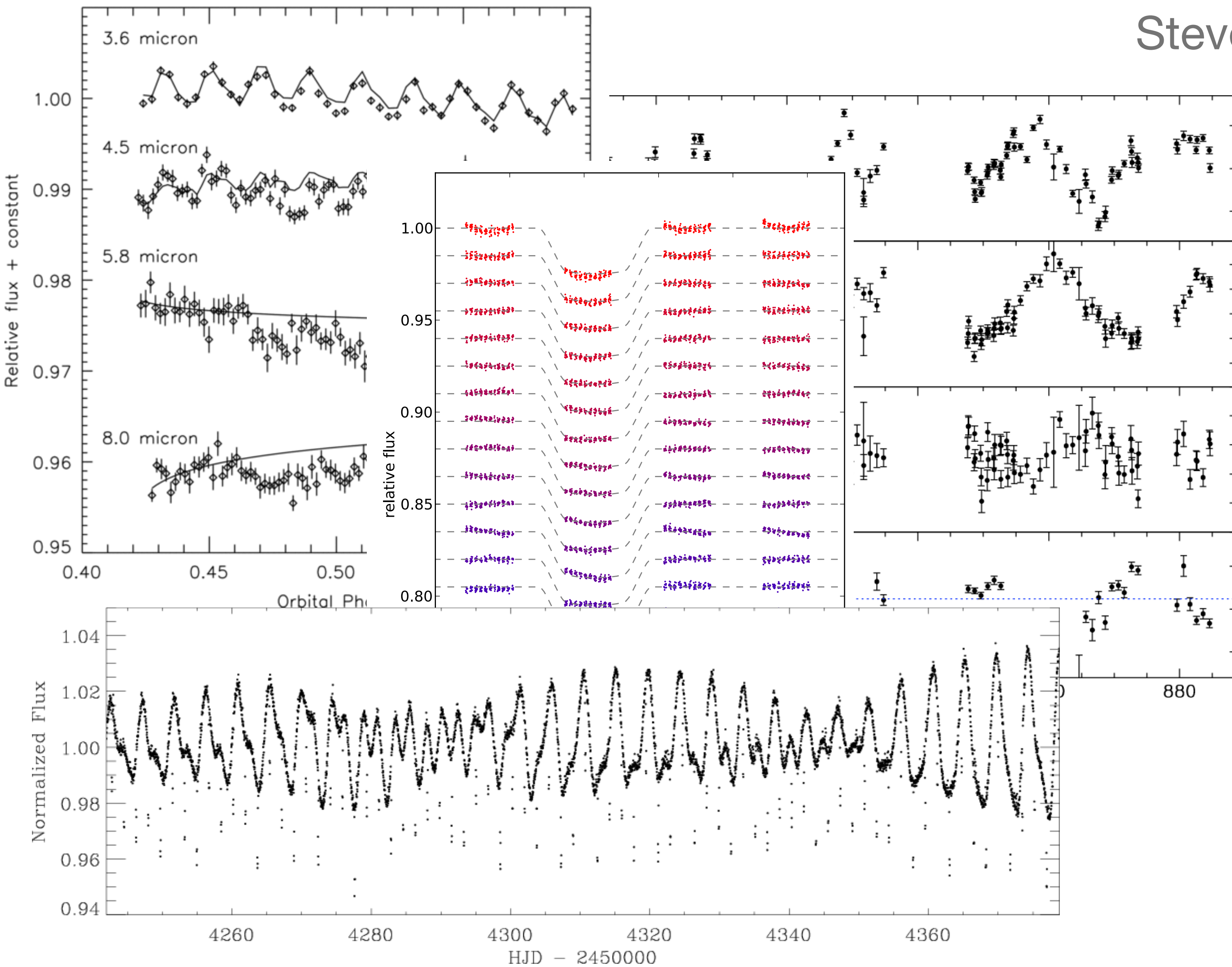

stochastic
high precision
time-series
correlated
disentangle
noise

... let the data speak

UNIVERSITY OF OXFORD

A Gaussian process in a nutshell

$$P\left(\boldsymbol{t} \,|\, \mathbf{X}, \boldsymbol{\theta}, \phi\right) = \mathcal{N}\left[m(\mathbf{X}, \phi), \mathbf{K}\right]$$

A Gaussian process in a nutshell

output vector (e.g. fluxes)   input matrix (e.g. time, pointing, detector temperature...)

likelihood

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(hyper-)parameters

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multivariate Gaussian

(hyper-)parameters

mean function

covariance matrix

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$$K_{nm} \equiv \mathrm{cov}\left[\boldsymbol{x}_n, \boldsymbol{x}_m\right] = k(\boldsymbol{x}_n, \boldsymbol{x}_m, \boldsymbol{\theta})$$

kernel function

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mean function   mean function parameters

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kernel function   kernel parameters

Gaussian process regression

Gaussian process regression

a priori knowledge

Gaussian process regression

a priori knowledge

- chose kernel & mean functions
- guess initial hyper-parameters

Gaussian process regression

a priori knowledge → chose kernel & mean functions → prior probability distribution over functions

a priori knowledge → guess initial hyper-parameters → prior probability distribution over functions

Gaussian process regression

a priori knowledge

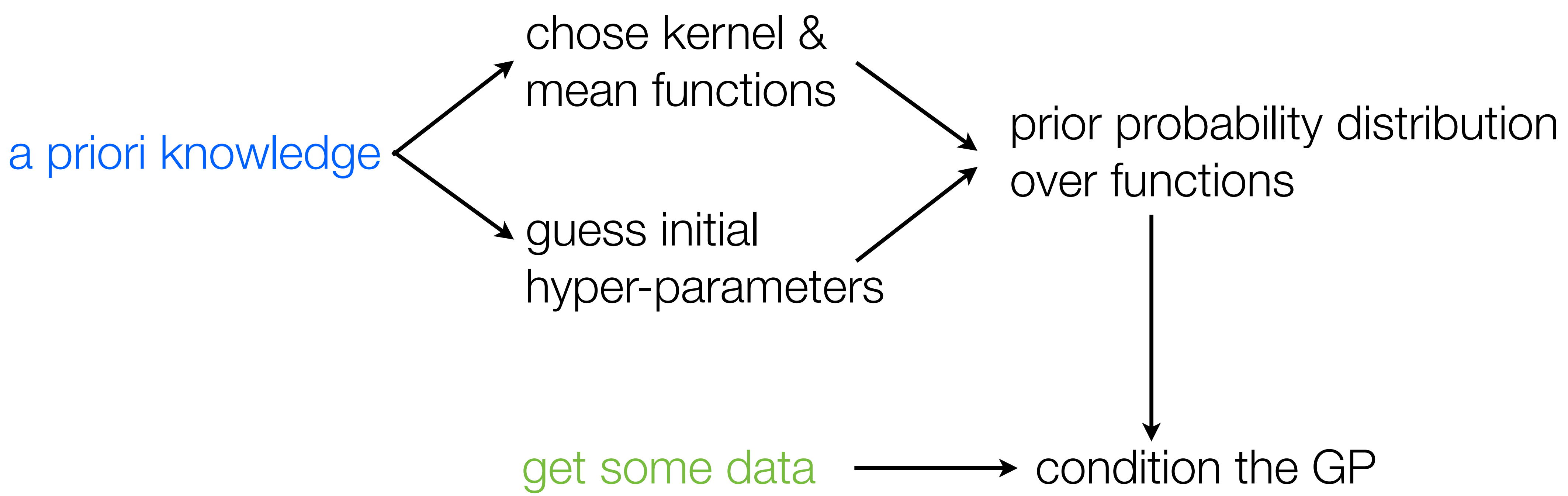
chose kernel & mean functions

guess initial hyper-parameters

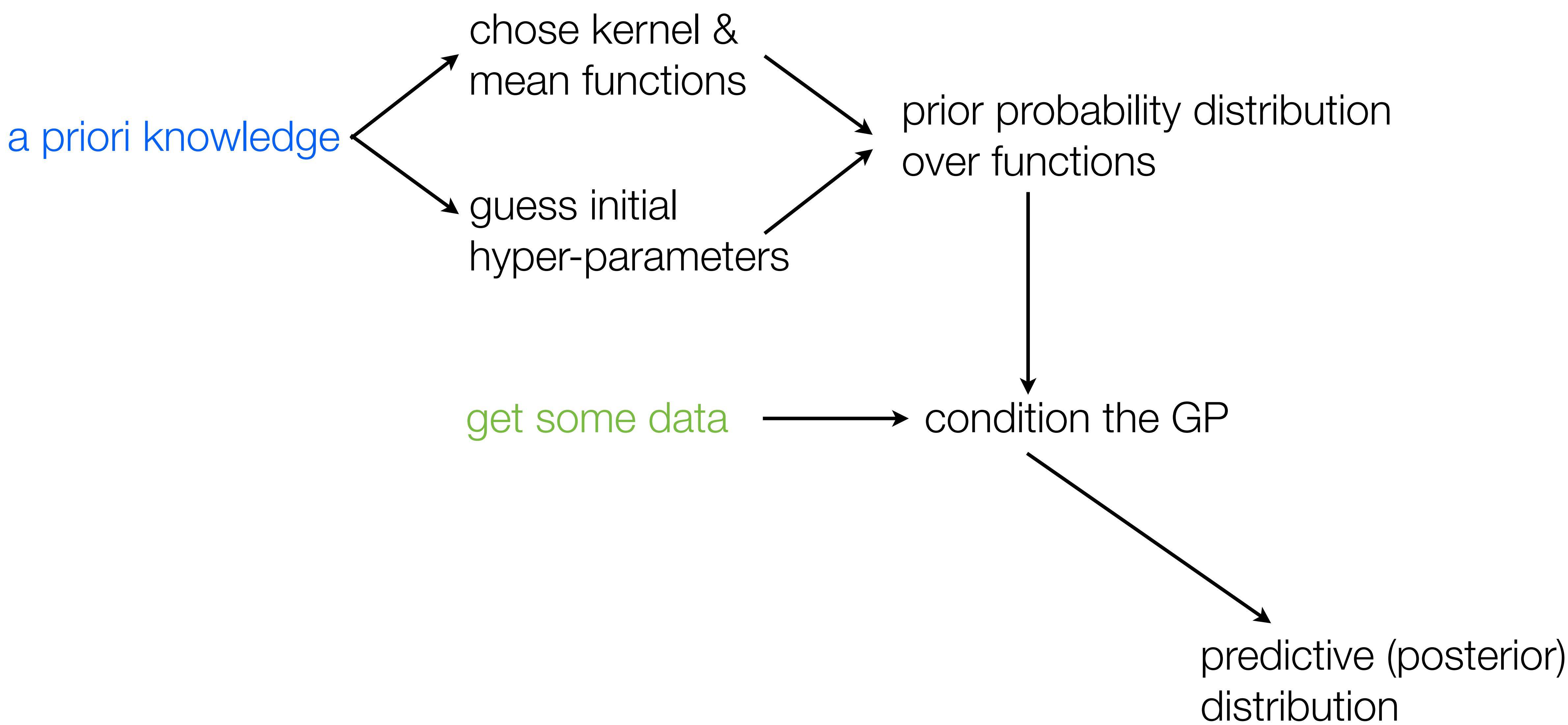
prior probability distribution over functions

get some data

Gaussian process regression



Gaussian process regression



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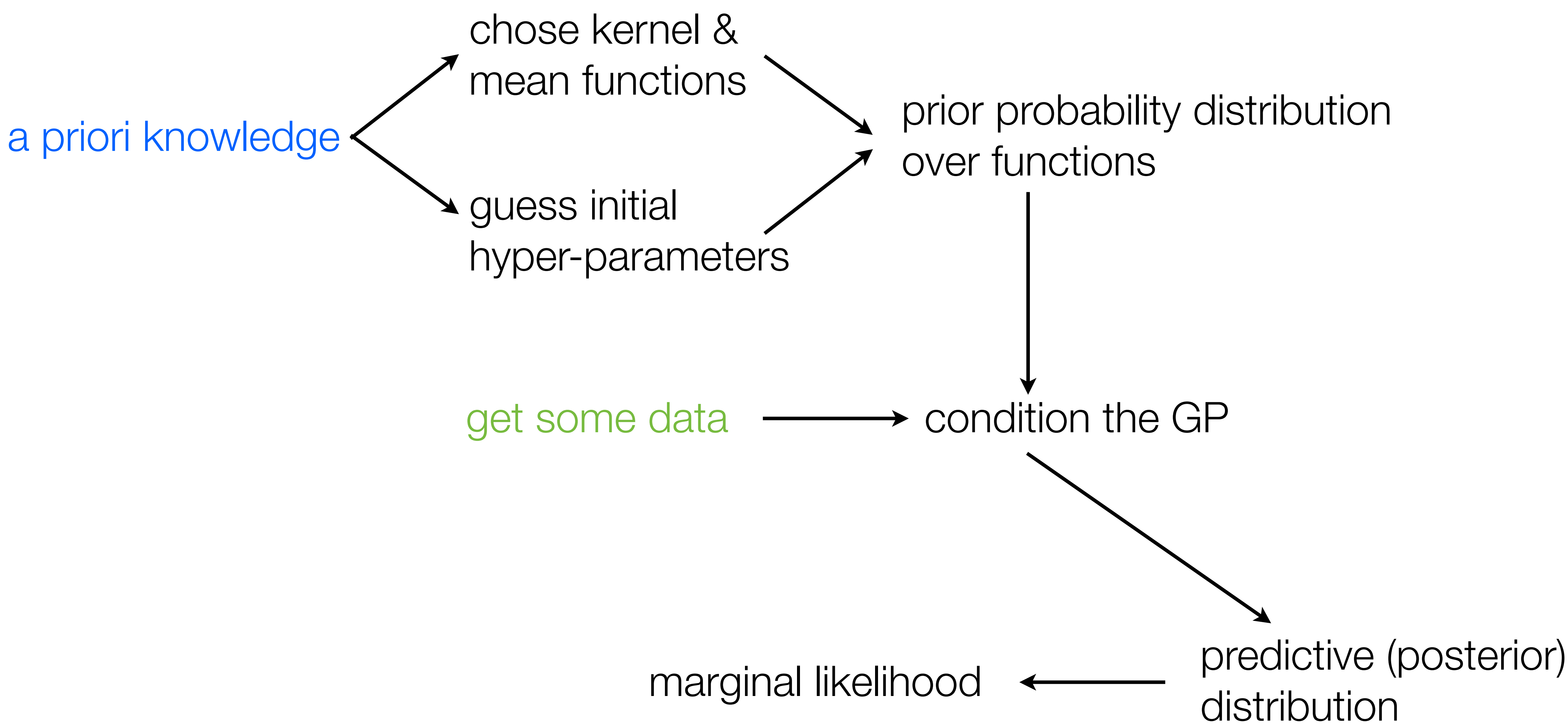

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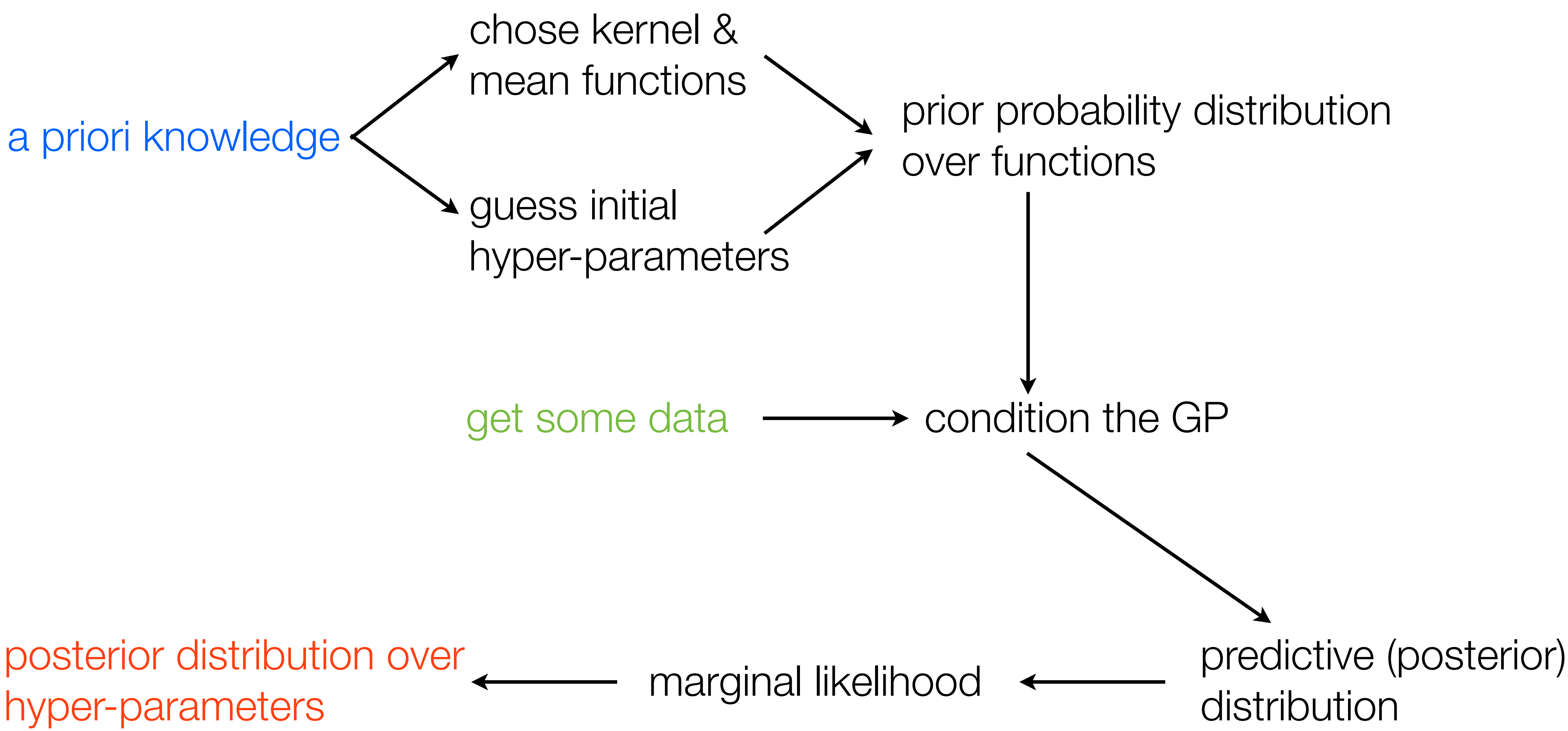

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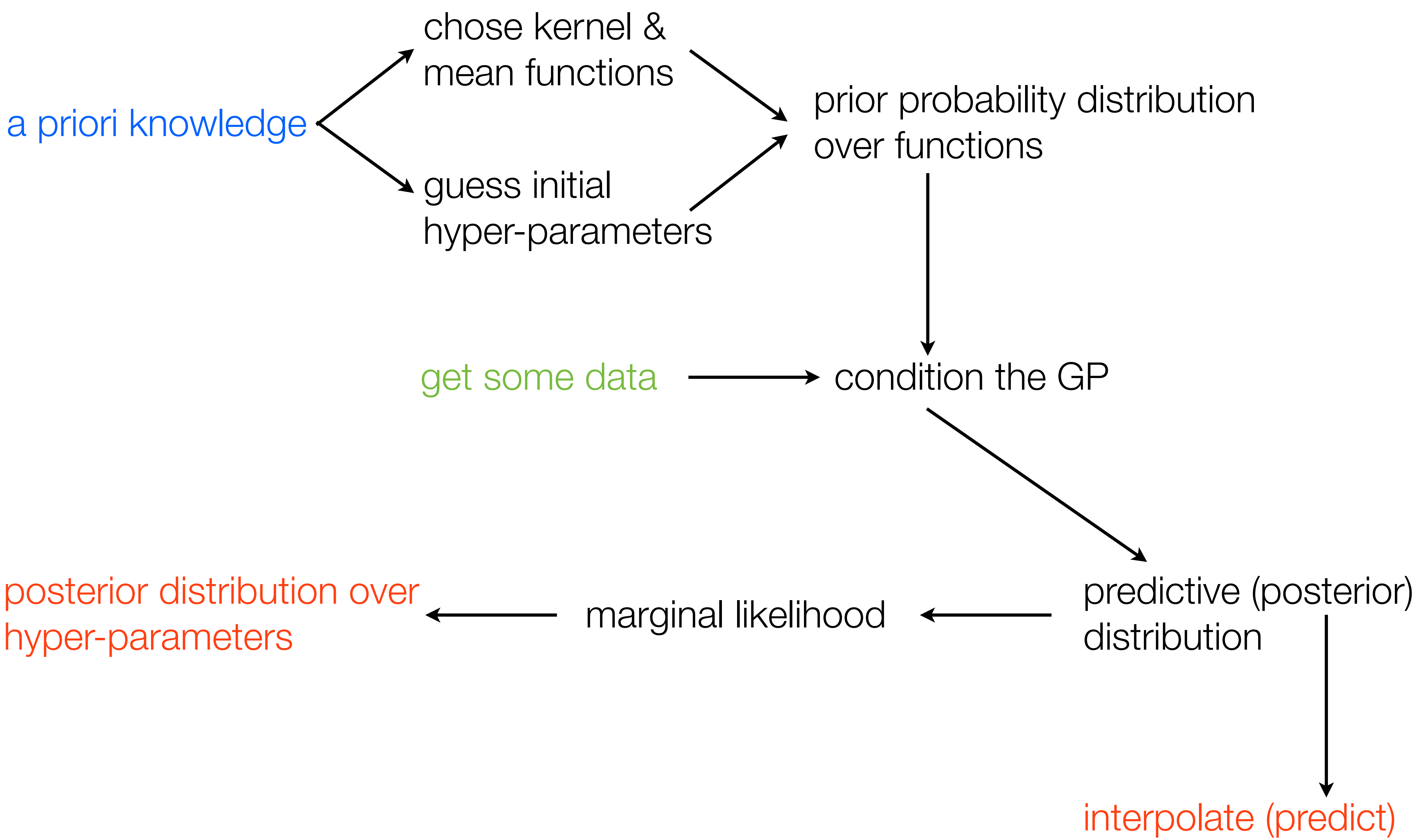
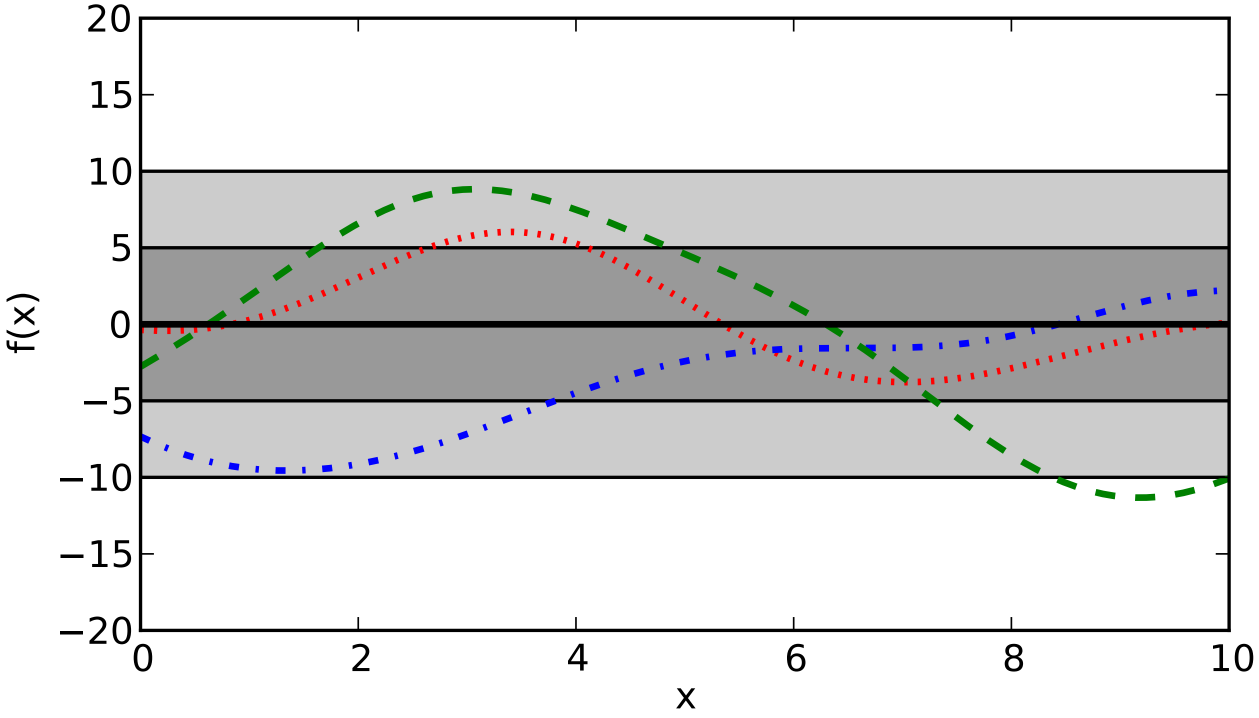
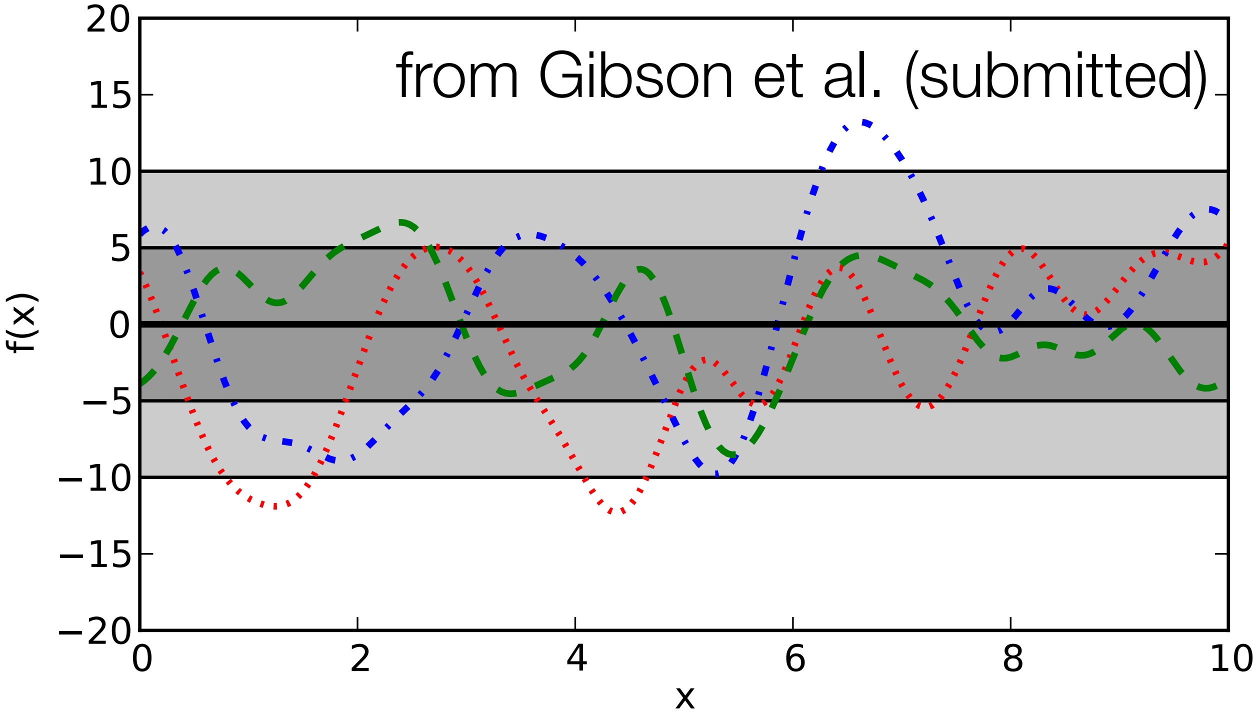


A very simple example

$$k_{\mathrm{SE}}(t, t') = A^2 \exp\left(-\frac{(t-t')^2}{2l^2}\right)$$





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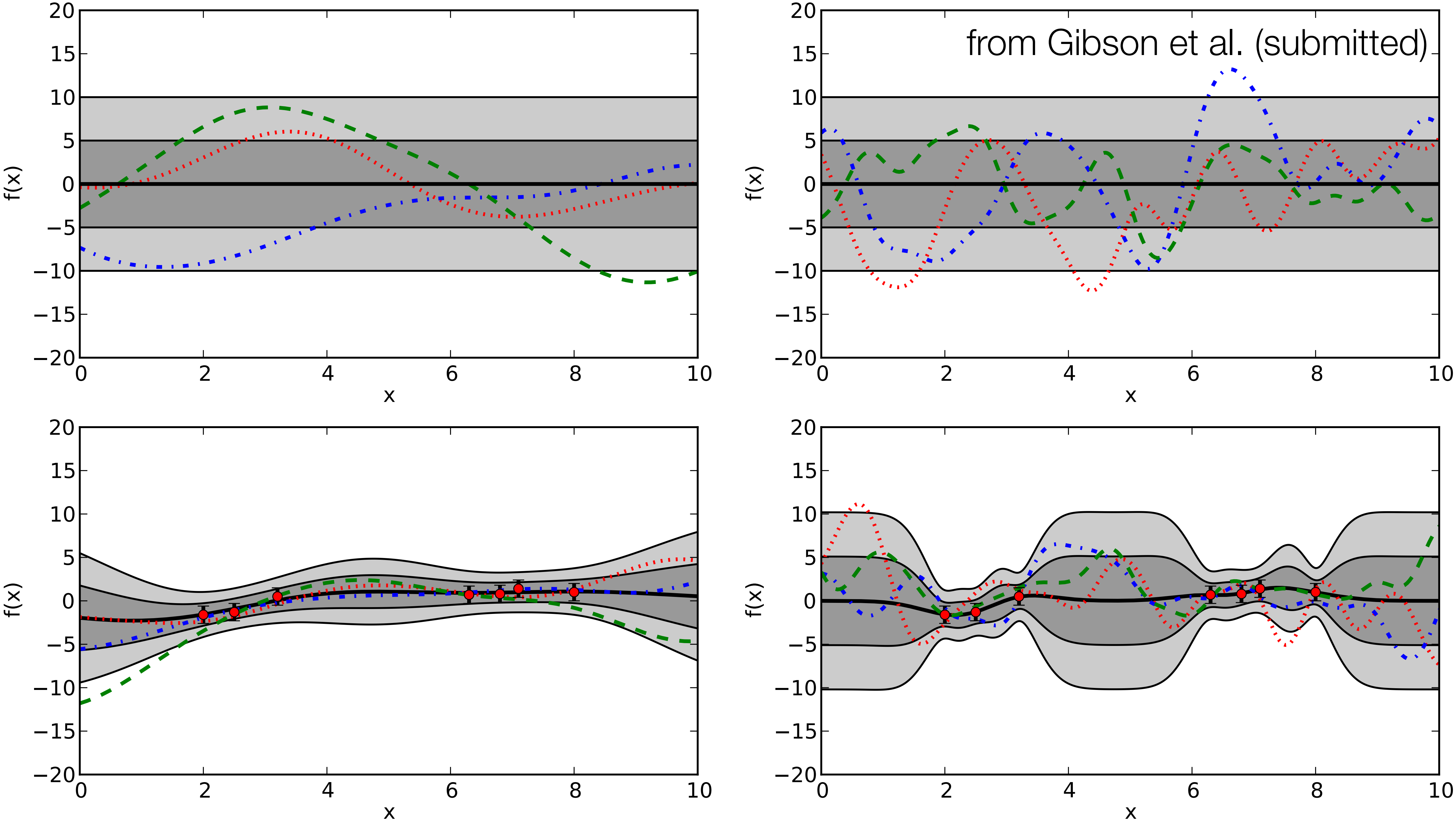

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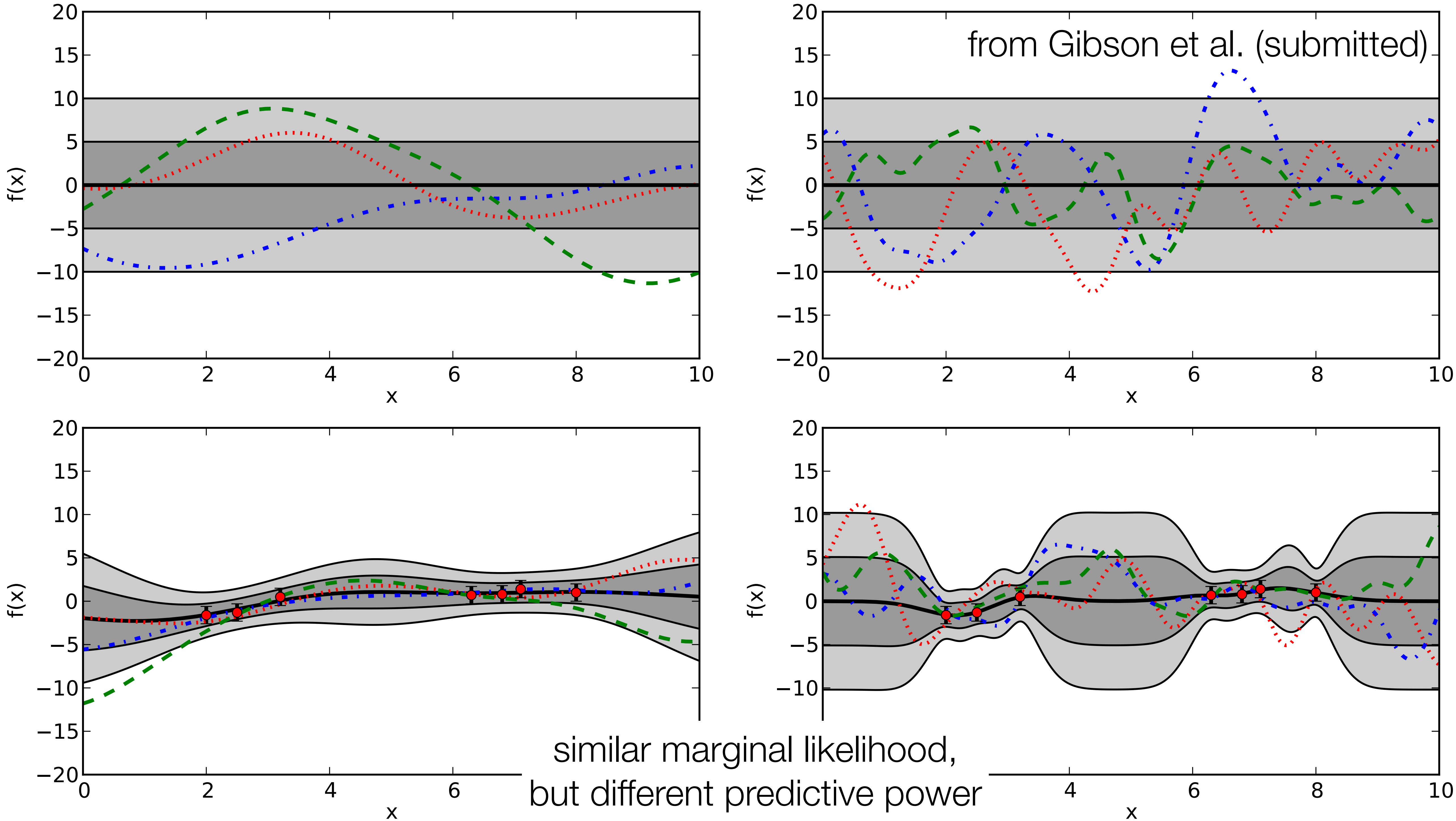
from Gibson et al. (submitted)



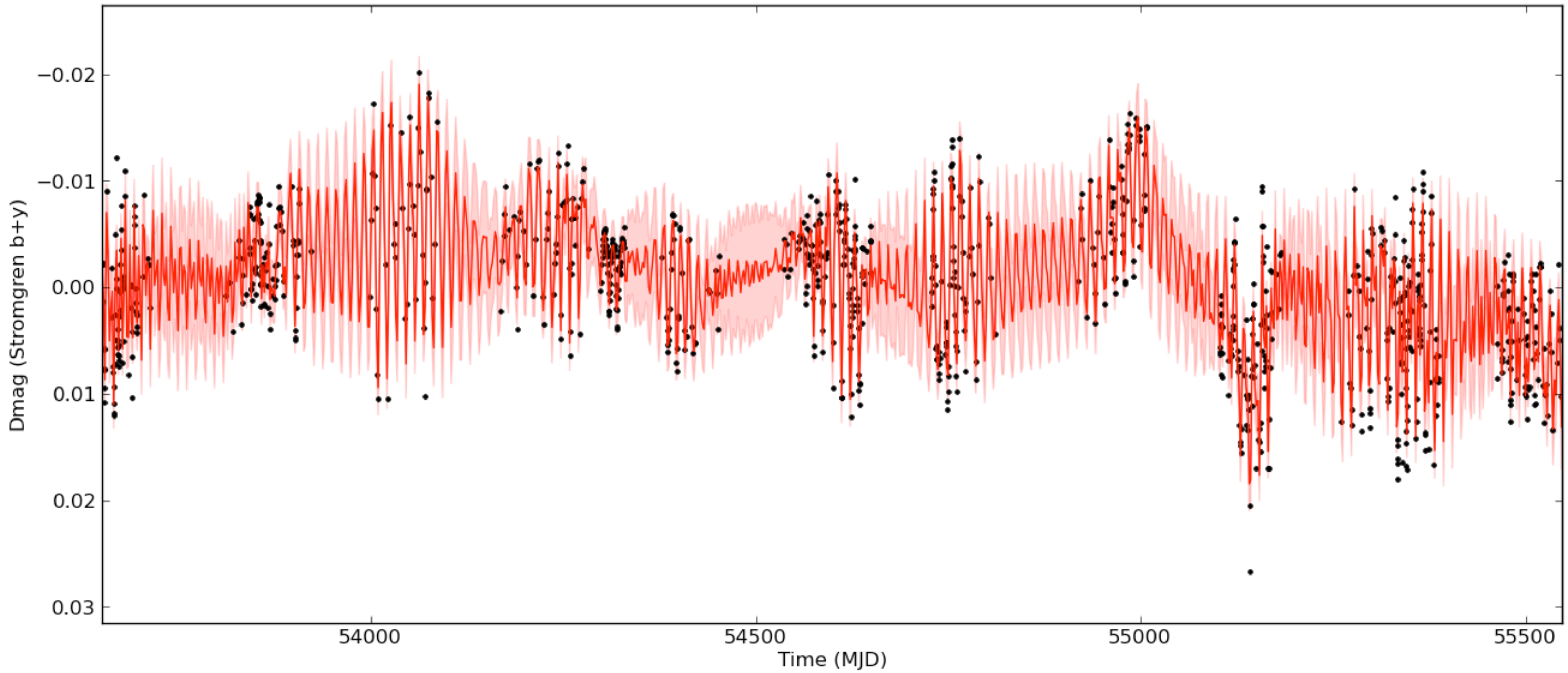
similar marginal likelihood,
but different predictive power

Example application 1:
instrumental systematics in transmission spectra

See Neale Gibson's talk

Example application 2: modelling HD 189733b's OOT light curve

$$k_{\mathrm{QP,mixed}}(t,t') = A^2 \exp\left(-\frac{\sin^2[\pi(t-t')/P]}{2L^2}\right) \times \left(1 + \frac{(t-t')^2}{2\alpha l^2}\right)^{-\alpha} + \sigma^2 \mathbf{I}.$$



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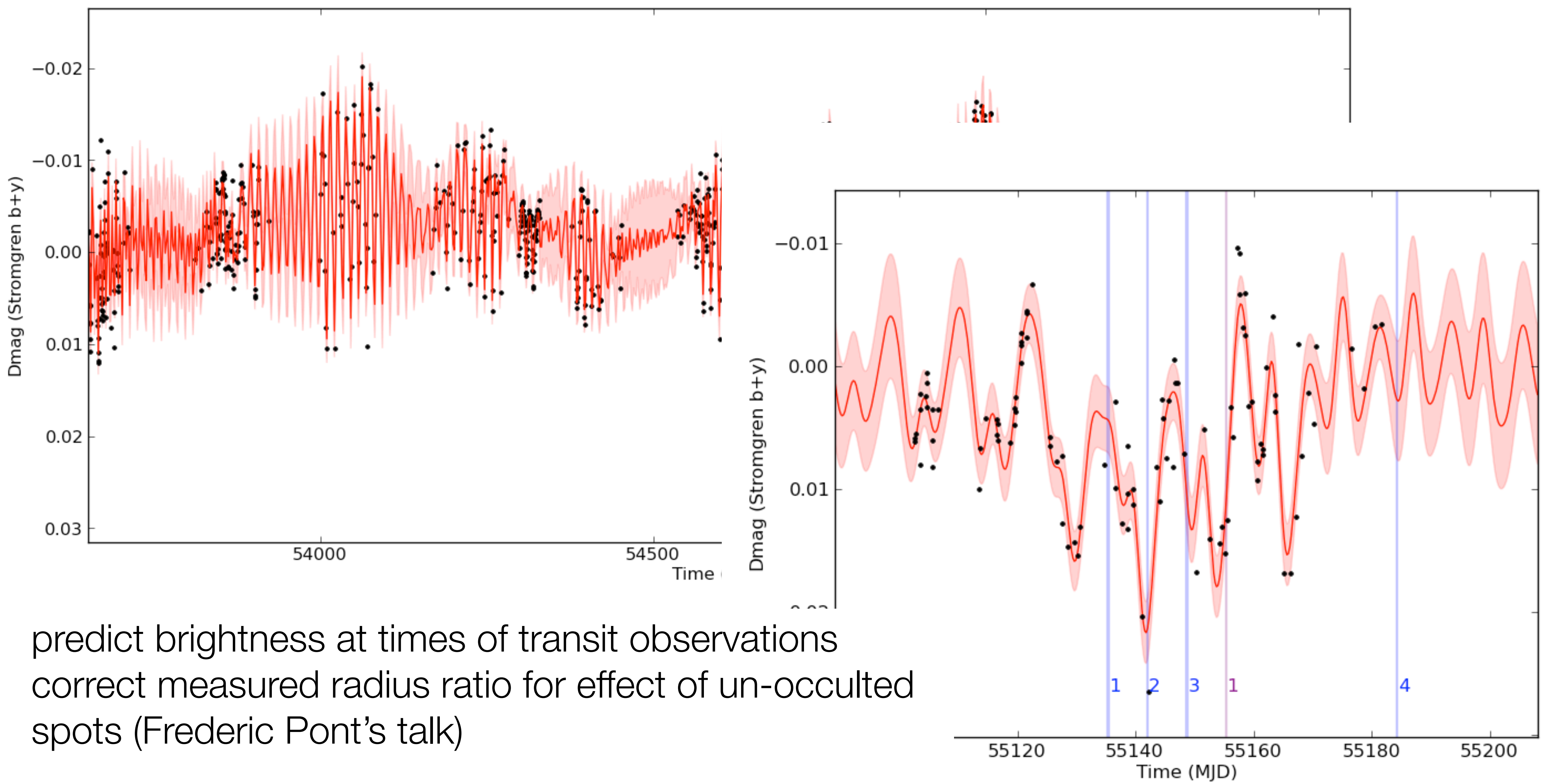

predict brightness at times of transit observations
correct measured radius ratio for effect of un-occulted spots (Frederic Pont's talk)

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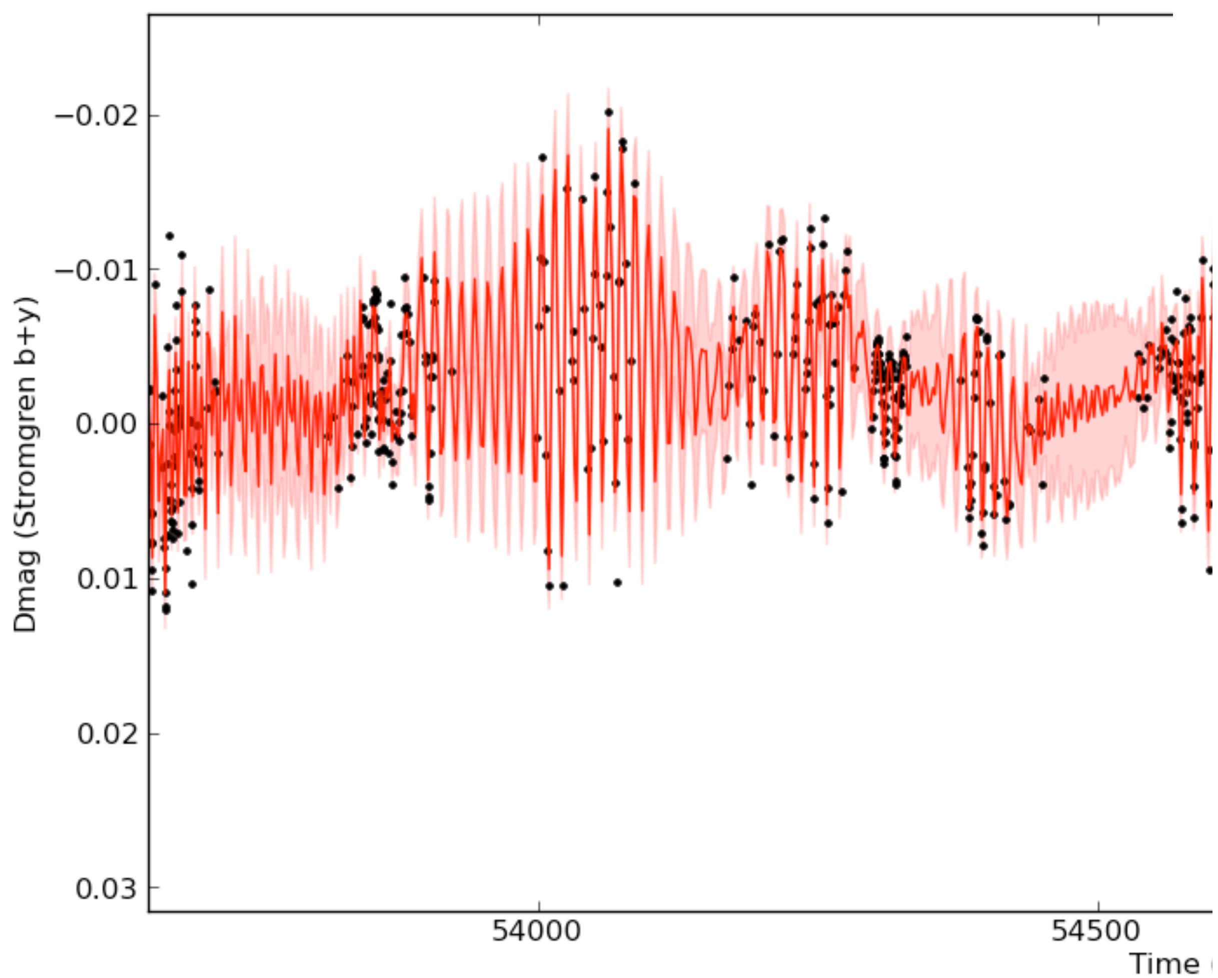
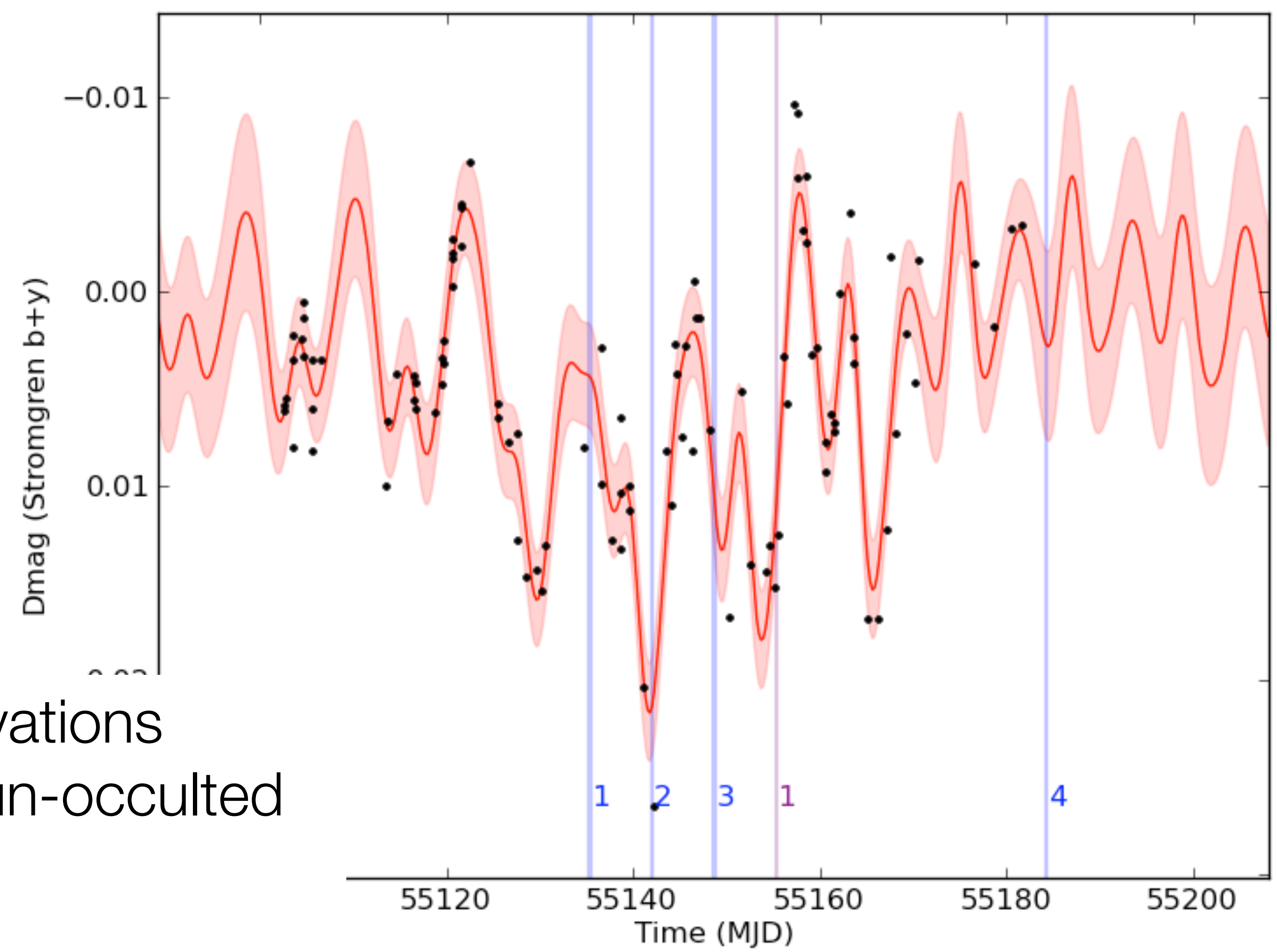


hyper-parameters contain information about rotation rate, spot distribution and evolution...



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Pros ... ... and cons

- Rigorous error propagation
- Extremely versatile
- Built-in Ockam's razor
- Joint modelling of arbitrary number of inputs (and outputs)
- Easy to combine with other techniques e.g. MCMC

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  - ok up to N~1000
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Want to try?
Python GP module under development