Steady state relativistic stellar dynamics around a massive black hole

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How do stars interact with, and fall into a massive black hole (MBH), and at what rates? (the "loss-cone" problem)

An analytic approach to loss cone dynamics with Monte Carlo solutions of the Fokker-Planck Eqn. in (E, J).

Physical processes

- Slow uncorrelated 2-body relaxation (NR).
- Fast coherent resonant relaxation [1] (RR).
  - Correlated background stellar torques.
  - Coherence time T_c.
- Secular Newtonian mass precession ω_M.
- Secular GR precession ω_{GR}.
- GR gravitational wave (GW) emission.

The loss-cone in phase-space

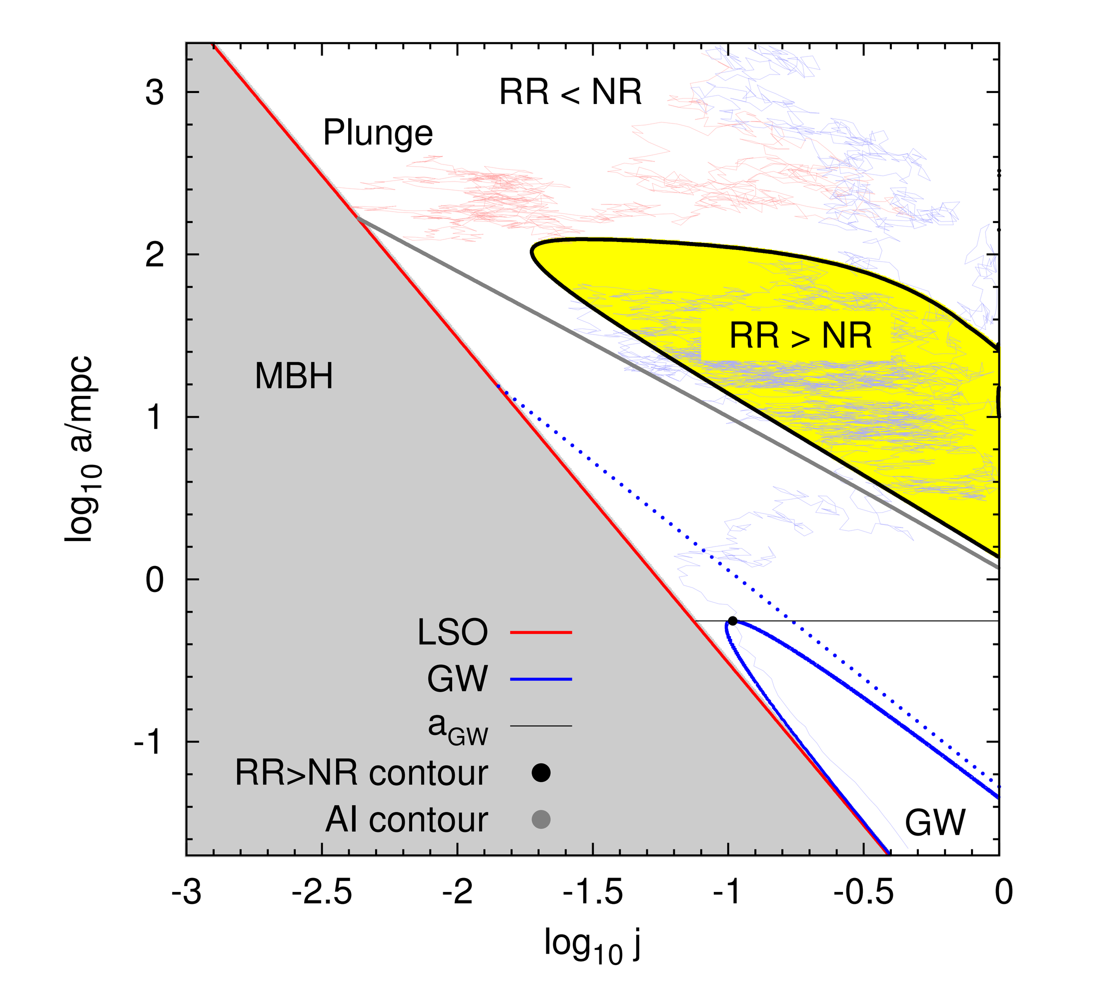

Figure 1: A schematic of the loss-cone phase space in semi-major axis (sma a) and normalized angular mom. ($j = \sqrt{1-e^2}$). Stars plunge into the MBH when they are scattered across the last stable orbit (red line), or spiral in by the emission of GW when they are scattered across the GW line (blue line), below the critical sma a_{GW} (the oft-assumed approximate GW line (blue dots) over-estimates the GW event rate). Adiabatic invariance suppresses RR below the AI line (gray), but RR is faster than NR only in the yellow region. Therefore, RR *does not* deliver stars all the way to the loss-lines (plunge or GW). The bottleneck remains slow NR [2].

Adiabatic Invariance (AI)

(aka "Schwarzschild Barrier" phenomenon [3])

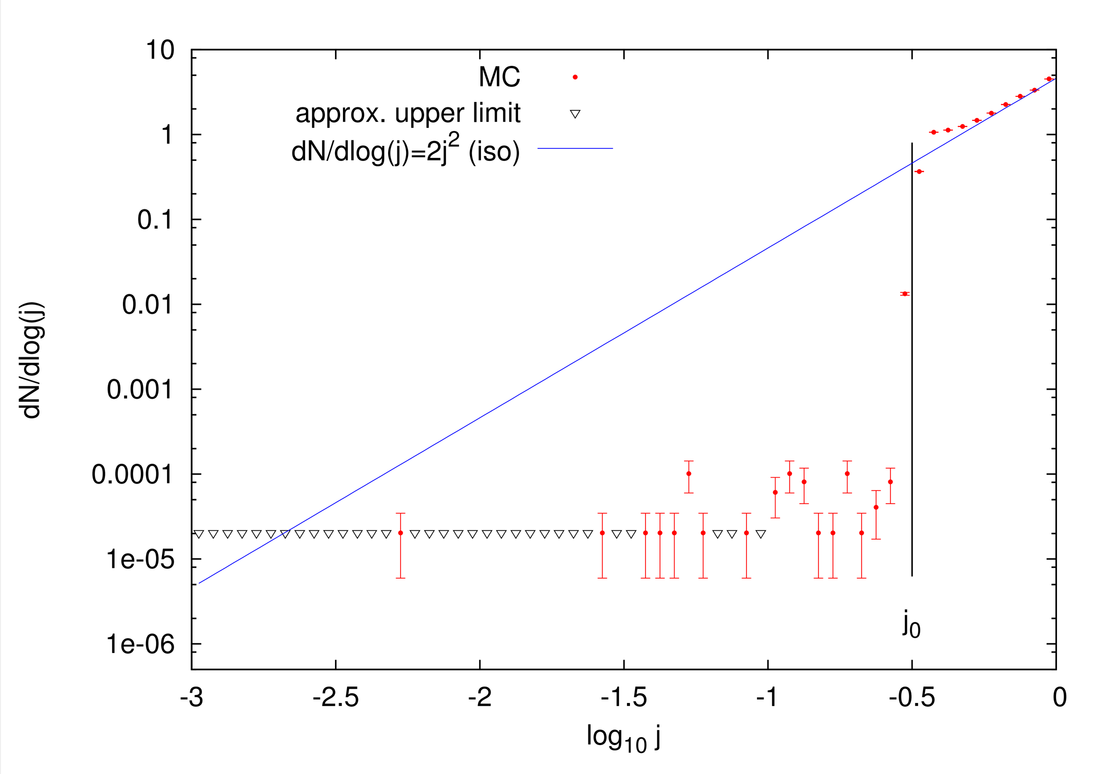

Figure 2: Adiabatic invariance in angular momentum due to *smooth* RR torque "noise" by the background stars [4]. The phase-space density below some small angular mom. $j_0 = \sqrt{T_c\omega_{GR}/2\pi}$ drops sharply when the GR precession period falls below the RR coherence time. In the absence of NR, the expected maximal entropy configuration (blue line) is reached only after an exponentially long time (i.e. never).

2-body relaxation: The great eraser

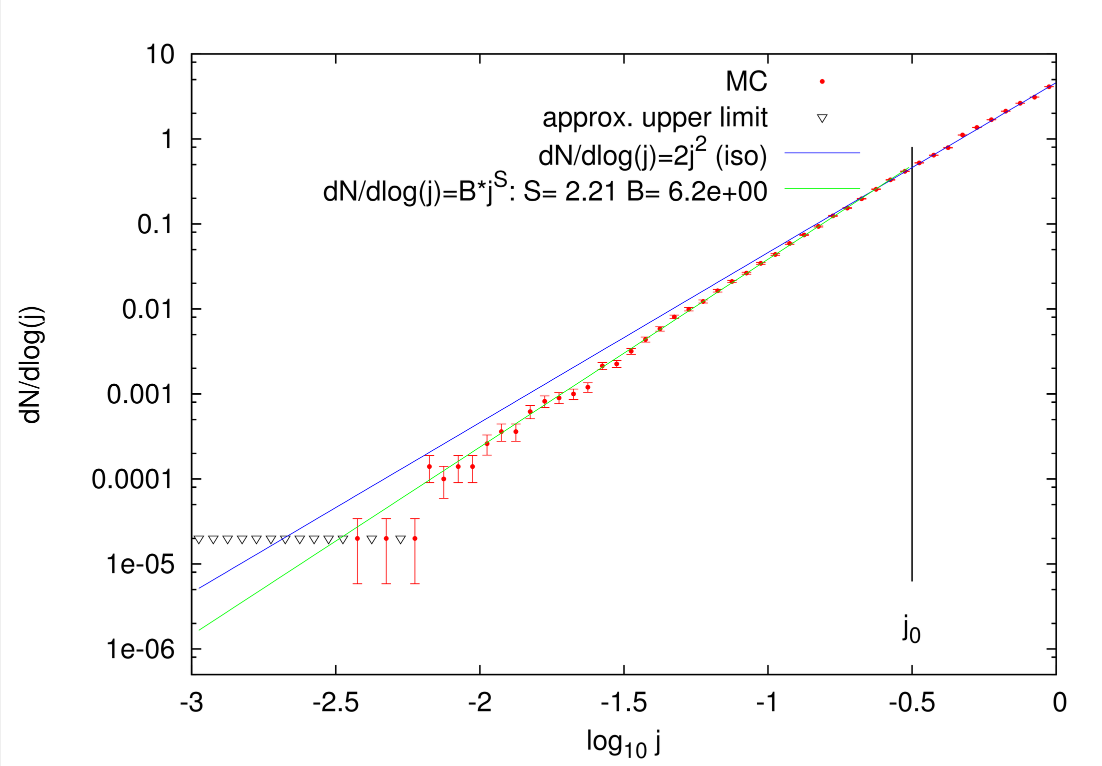

Figure 3: NR erases the AI barrier almost completely in one 2-body relaxation timescale. The phase-space density approaches the maximal entropy configuration (blue line) in the long timescale (steady state) limit.

Effective RR diffusion coefficients (DCs) that incorporate the correlated noise and the secular precessions [4], together with NR DCs and GW dissipation, provide a powerful scalable Monte Carlo (MC) tool for modeling the dynamics and loss-rates of galactic nuclei.

Comparison with direct N-body

$N = 50$ "cusp" rates in Myr^{-1} (*N*-body *not* in steady state!)	MC	NB [3]	NB [5]
Plunge	0.7	0.2	0.5±0.1
Inspiral	1.4	0.9	0.8±0.1

Loss-rates from MW-like galaxies

$N \sim \mathcal{O}(10^6)$, $M_\star = 10 M_\odot$ stellar BHs

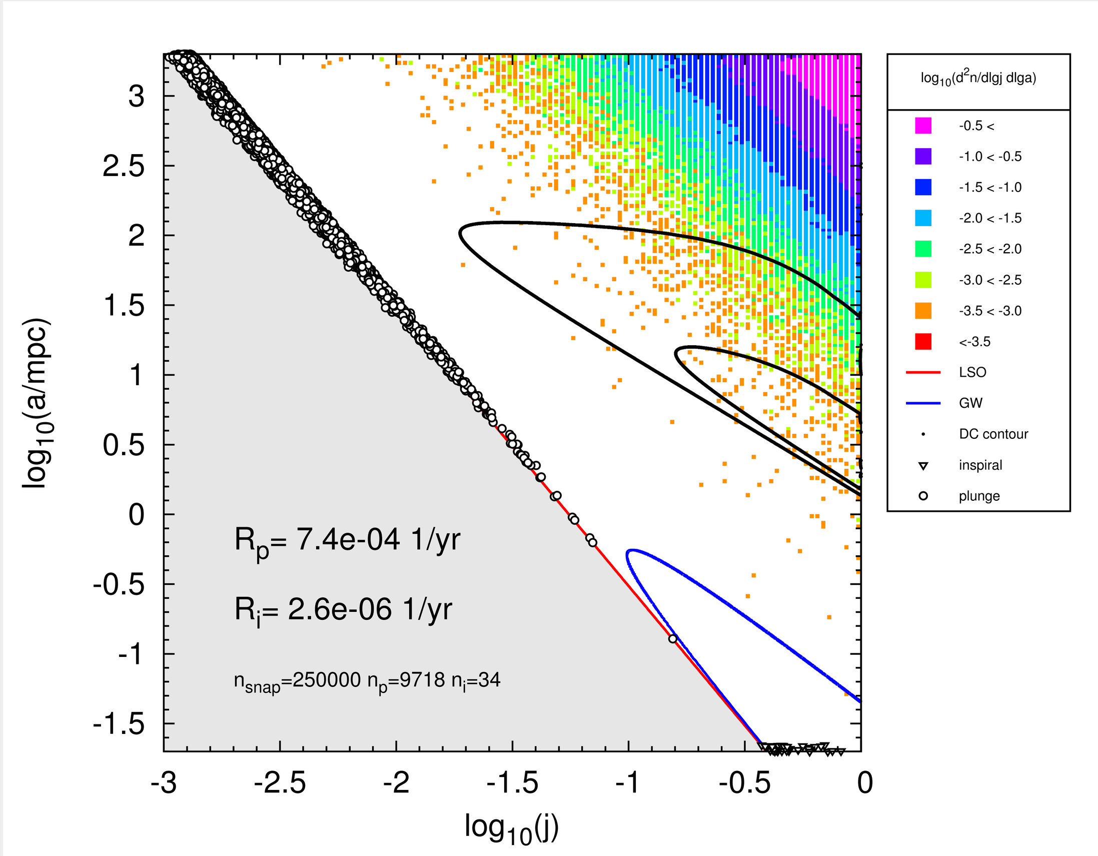

Figure 4: The phase-space density, plunge (R_p) and GW inspiral (R_i) loss-rates for a MC model of the Milky Way (MW).

Loss-rate scaling with MBH mass

The analytic model agrees well with the MC rates and published approximate power-law fits (Figure 5). E.g. for relaxed stellar cusps with $n_\star \propto r^{-7/4}$ following an $M_\bullet \propto \sigma^4$ $M_\bullet/\sigma$ relation:

$$R_p \propto Q^{-1/4} \log Q \quad \text{and} \quad R_i \propto Q^{-1/4} (\log Q)^{1/5}$$

RR can matter (example): Disruption of red giants captured by binary tidal separation (Figure 6).

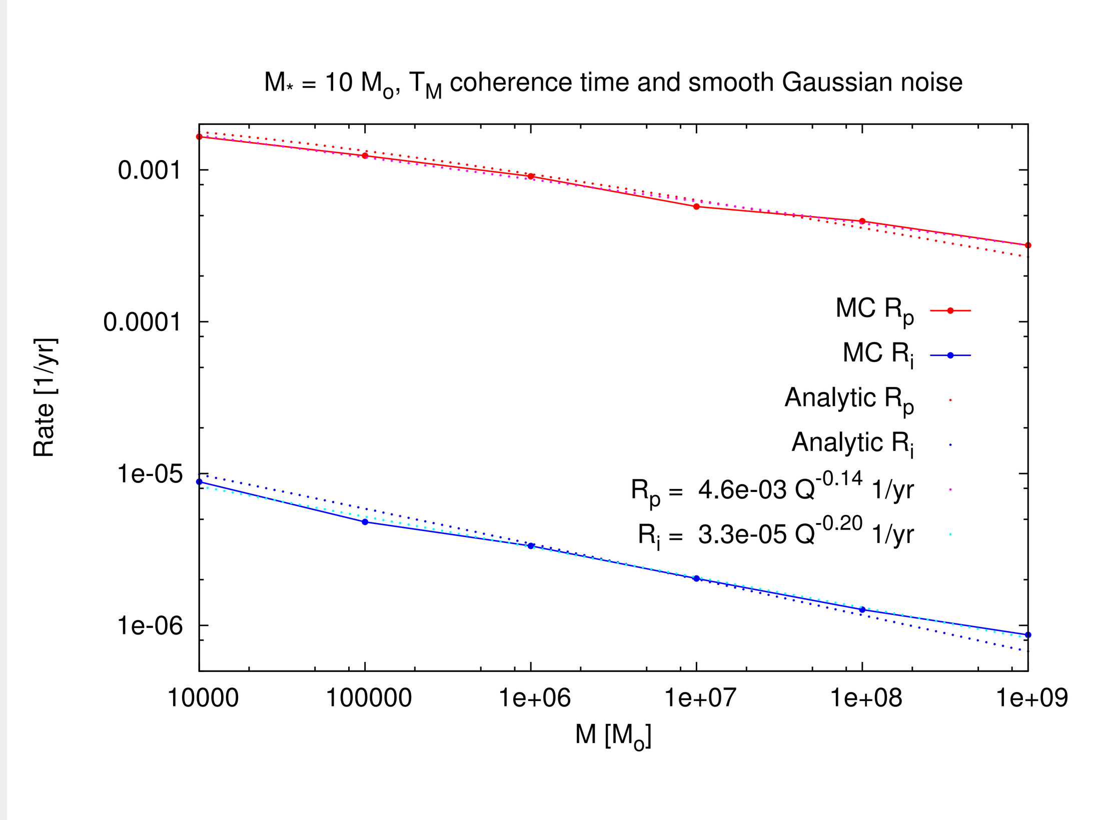

Figure 5: The scaling of the loss-rates with $M_\bullet$.

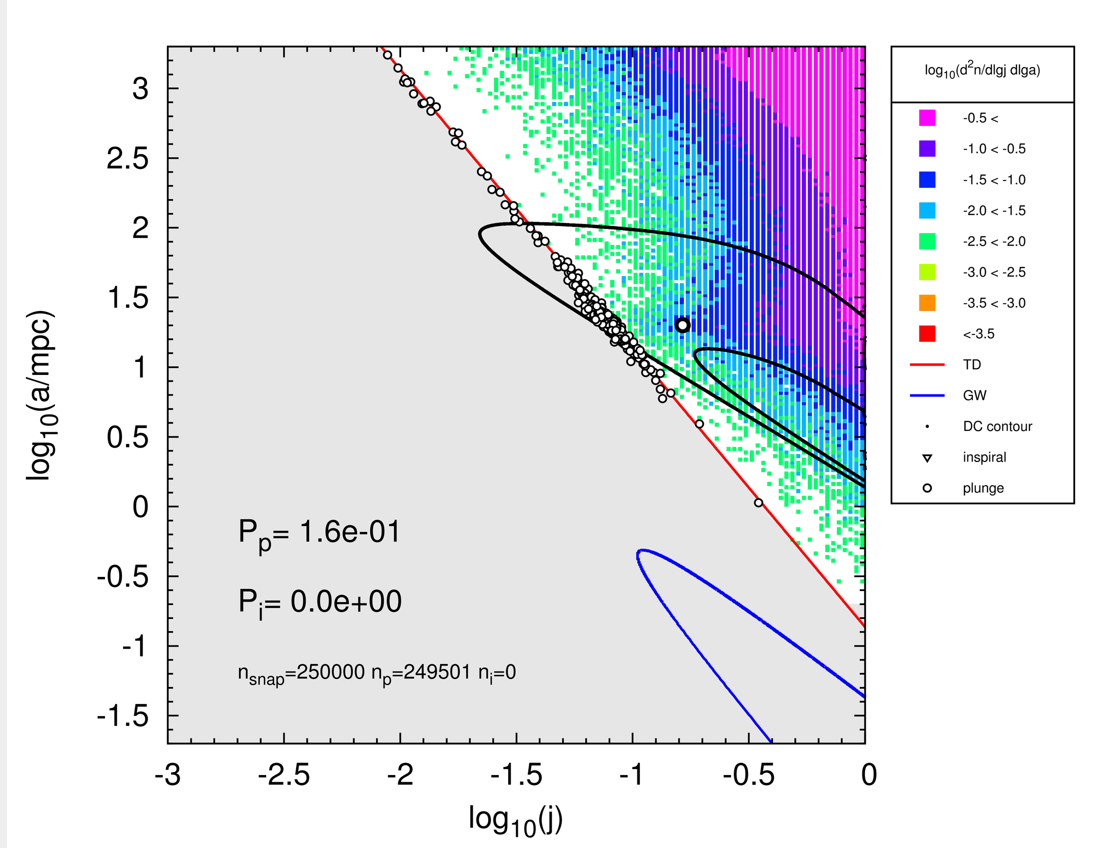

Figure 6: The branching ratio P_p for the tidal disruption of red giants that were Hills-captured (at white circle), is $\sim \times 3$ higher due to RR, since the large tidal disruption loss-line intersects the RR-dominated region.

Summary: NR, RR, GW dissipation and secular precession can be treated analytically as an effective diffusion process. Long-term steady state depends mostly on NR, which erases AI. RR can be important in special cases.

References

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