Tidal dissipation in convective regions of planets and stars

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Jackson Hole  14.09.11
Effects of tidal dissipation

Dissipation in the star:
- orbital decay
- spin-orbit alignment

Dissipation in the planet:
- spin synchronization
- orbital circularization
- heating

Tidal theory:
- determine the rates of these processes
- how much dissipation (or torque) is produced when a body is forced by a potential $\propto r^l Y_{lm}(\theta, \phi) \exp(-i\omega t)$?
  (depends on frequency $\omega$ and quantum numbers $l, m$)
Tides in convective regions of planets and stars

- Turbulent viscosity acting on tidal bulge?
- Excitation and dissipation of inertial waves?

[ For radiative regions see poster 34.03 by Adrian Barker ]
Linear tides in barotropic fluid bodies

- Barotropic: no stable stratification or internal gravity waves
- Low-frequency tides in slowly rotating bodies:
  \[ \omega \sim \Omega \sim \epsilon \left( \frac{GM}{R^3} \right)^{1/2}, \quad \epsilon \ll 1 \]

- Systematic theory based on expansion in powers of \( \epsilon^2 \)
- Displacement \( \xi = \xi_{nw} + \xi_w \)
- Non-wavelike part:
  response of spherical body to tidal potential neglecting Coriolis
  (easily computed but different from classical equilibrium tide)
- Wavelike part:
  residual response (inertial waves)
  known body force from Coriolis force on non-wavelike part
Periodic forcing of inertial waves

- Consider inertial waves driven by body force $\propto \exp(-i\omega t)$
  deriving from tidal potential $\propto r^l Y^m_l(\theta, \phi) \exp(-i\omega t)$
- Calculate linear response with same frequency

Selected references:

Ogilvie & Lin (2004)
Wu (2005)
Ogilvie & Lin (2007)
Ivanov & Papaloizou (2007, 2010)
Goodman & Lackner (2009)
Ogilvie (2009)
Rieutord & Valdettaro (2010)
Idealized problem: isentropic rotating fluid in spherical geometry

- Rigid core, fractional radius 0.5

\[ l = m = 2 \]

Typical results

decreasing viscosity
Typical results

- Caveats:
  - convective background
  - magnetic fields
  - reflections
  - nonlinear breakdown
Impulsive forcing of inertial waves

- Consider inertial waves driven by body force $\propto \delta(t)$
- Impulsive response is smooth and readily computable (ODEs)
- Will subsequently resolve into complicated pattern of inertial waves which may dissipate through linear or nonlinear processes
- (Kinetic) energy of impulsive response gives a broad frequency average of the response function
- Equivalent to frequency-average of tidal time lag, or $\frac{1}{\omega Q}$
- Robust result, dependent only on gross structure
- Also more directly applicable to highly eccentric orbits
Impulsive energy transfer / frequency-averaged dissipation

- Homogeneous fluid body with rigid core
  - Sectoral harmonics $m = l$

  $$\hat{E} \propto \frac{\alpha^{2l+1}}{1 - \alpha^{2l+1}} \quad \alpha = \frac{r_{\text{in}}}{R}$$

- Tesseral harmonics $m < l$

  $$\hat{E} \propto \frac{1}{1 - \alpha^{2l+1}} \quad (+ \text{term as above})$$

but beware trivial inertial modes with $l = 2$
Impulsive energy transfer / frequency-averaged dissipation

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    \hat{E} \propto \frac{1}{1 - \alpha^{2l+1}} \quad (+ \text{term as above})
    \]
    but beware trivial inertial modes with \( l = 2 \)

- Two homogeneous fluids
  - Similar but weaker result
  - Strengthened if densities differ greatly
Impulsive energy transfer / frequency-averaged dissipation

- Polytrope with rigid core

\[ p \propto \rho^{1+1/n} \]

\[ n = 0 \text{ (homogeneous)} \]
\[ n = 1 \]
\[ n = 3 \]
\[ n = 0.3 \]
\[ n = 0.1 \]
\[ n = 0 \text{ (homogeneous)} \]

\[ \propto r_{\text{in}}^5 \]

\[ l = m = 2 \]

\[ \hat{E} \]

impulse energy

\[ r_{\text{in}}/R \]
Effective viscosity of turbulent convection

- How does a convecting fluid respond to periodic distortion?
- Oscillatory shearing box

\[ a \sin(\omega t) \]
Effective viscosity of turbulent convection

- Compute convective or other flow in OSB
- Measure Reynolds stress at frequency $\omega$

$a \sin(\omega t)$
Previous hypotheses and results

- Zahn (1966): viscosity $\propto \omega^{-1}$ (large eddies)
- Goldreich & Nicholson (1977): viscosity $\propto \omega^{-2}$ (small eddies)
- Goodman & Oh (1997): viscosity $\propto \omega^{-5/3}$ (small eddies)
- Penev et al. (2009): viscosity $\propto \omega^{-1}$

![Graph showing the relationship between Forcing Period/Convective Turnover Time and $K_{1212}^0$]
Convection in an oscillatory shearing box (Geoffroy Lesur)
Convection in an oscillatory shearing box (Geoffroy Lesur)

- Time series of Reynolds stress (shear stress)
Convection in an oscillatory shearing box (Geoffroy Lesur)

- Fourier transform of Reynolds stress (shear stress)
Convection in an oscillatory shearing box (Geoffroy Lesur)

- Real part of effective viscosity versus tidal frequency

\[ \text{numerical simulation} \]
\[ \text{closure model} \]

\[ \text{uncertainty due to noise} \]

\[ \propto \omega^{-2} \]

(dashed: negative)
Imaginary part of effective viscosity versus tidal frequency

Numerical simulation closure model

\[ \propto \omega^{-1} \]
Analytical results for high-frequency shear

General flow (laminar, turbulent, convective, ...)

Tidal period $\ll$ flow timescales (relevant for large eddies)

- Dominant response is elastic
- Next effect is viscosity $\propto \omega^{-2}$
- Coefficients may be positive, negative or zero depending on flow statistics, anisotropy, etc.
- Incompatible with Zahn (1966)
- Different from Penev et al. (2009)
- Raises the possibility of tidal anti-dissipation
Conclusions

- Idealized response (inertial waves) is highly frequency-dependent
- Frequency-averaged dissipation is robust and readily calculated
- For \( l = m = 2 \) dissipation is most efficient for:
  - larger, more rigid or denser cores
  - greater density stratification (larger polytropic index)
- Other (tesseral) harmonics excite richer response and may be important even though intrinsically weaker

- High-frequency tidal response of convection (and other flows) is:
  - elastic (+, – or 0)
  - viscous (+, – or 0), \( \nu \propto \omega^{-2} \) and therefore small
  - anisotropic
- More work required for stellar or planetary application