

New Limits on the Strong Equivalence Principle from Two Long-period Circular-orbit Binary Pulsars

D. R. Lorimer

University of Manchester, Jodrell Bank Observatory, SK11 9DL, UK

P. C. C. Freire

NAIC, Arecibo Observatory, HC3 Box 53995, Arecibo, PR 00612, USA

Abstract. Following a brief review of the principles of the strong equivalence principle (SEP) and tests for its violation in the strong and weak gravitational field regimes, we present preliminary results of new tests using two long-period binary pulsars: J0407+1607 and J2016+1947. PSR J0407+1607 is in a 669-day orbit around a $\gtrsim 0.2 M_{\odot}$ companion, while J2016+1947 is in a 635-day orbit around a $\gtrsim 0.3 M_{\odot}$ companion. The small eccentricities of both orbits ($e \sim 10^{-3}$) mean that these systems reduce previous limits on SEP violation by more than a factor of 4.

1. Equivalence Principles and Gravitational Self-energy

The principle of equivalence between gravitational force and acceleration is a common feature to all viable theories of gravity. The Strong Equivalence Principle (SEP), however, is unique to Einstein's general theory of relativity (GR). Unlike the weak equivalence principle (which dates back to Galileo's demonstration that all matter free falls in the same way) and the Einstein equivalence principle from special relativity (which states that the result of a non-gravitational experiment is independent of rest-frame velocity and location), the SEP states that free fall of a body is completely independent of its gravitational self energy. Before examining how the SEP can be tested, let us first review the gravitational self energy, ϵ , which is a useful quantity for distinguishing between strong or weak gravitational fields. Expressed in terms of the rest-mass energy of a body of mass M and size R , $\epsilon = -GM/Rc^2$. For most bodies, ϵ is vanishingly small. For example $\epsilon_{\text{human}} \sim -10^{-26}$, $\epsilon_{\text{Earth}} \sim -5 \times 10^{-10}$ and even $\epsilon_{\odot} = -2 \times 10^{-6}$. Only for compact objects does ϵ become significant and we enter the "strong-field" regime. For a white dwarf $\epsilon_{\text{WD}} \sim -10^{-4}$, for a neutron star $\epsilon_{\text{NS}} \sim -0.3$ and for a non-rotating black hole, $\epsilon_{\text{BH}} = -0.5$.

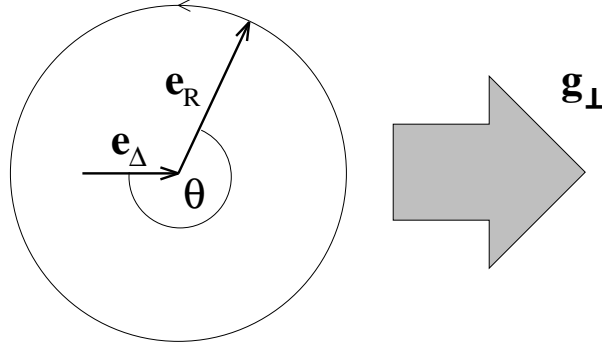


Figure 1. Diagram from Wex (1997) showing two competing effects on the orientation of a binary system’s eccentricity: the component in the direction of the local gravitational field due to an SEP violation (e_{Δ}) and the rotation due to relativistic periastron advance (e_R).

2. Testing the Strong Equivalence Principle

If the SEP is violated, then the ratio of inertial mass (m_i) to gravitational mass (m_g) of a test particle differs from unity by an amount

$$\frac{m_i}{m_g} - 1 \equiv \Delta = \eta\epsilon + \eta'\epsilon^2 + \dots, \quad (1)$$

where η parametrises the violation in terms of ϵ . A violation of the SEP ($\Delta \neq 0$) would mean that two bodies of unequal self-energies fall differently in an external gravitational field. If these bodies were in orbit about one another, their differential free fall would cause the orbit to be “polarized” in the direction of the external field. This so-called “Nordvedt effect” was first proposed as a test of the SEP for the Earth-Moon system in the Sun’s gravitational field (Nordvedt 1968a,b). While lunar laser ranging measurements of the Nordvedt effect place fairly stringent constraints ($|\eta| < 0.0016$; see Will 2001 for a review), since ϵ_{Earth} and ϵ_{Moon} are so small, this test only applies to the weak-field regime. Damour & Schäfer (1991) proposed a further means to test the SEP by an analogy of the Nordvedt effect on Galactic neutron star-white dwarf binary systems. In this case, the external gravitational field is provided by the Galaxy rather than the Sun and, as shown in Figure 1, for a violation of the SEP polarizes the eccentricity in the direction of the local gravitational field. Due to the larger and significantly different self energies of neutron stars and white dwarfs over solar system bodies, these binaries test the SEP in the strong-field regime.

We shall leave it as an exercise to the interested reader (see Damour & Schäfer 1991 for details) to show that the constraint provided by a binary system is

$$\Delta \leq \frac{8\pi^2 G M e \xi(\theta)}{g c^2 P_b^2 [1 - (\cos i \cos \lambda + \sin i \sin \lambda \sin \Omega)^2]^{1/2}}. \quad (2)$$

Here M is the total mass of the system, e is the orbital eccentricity, P_b is the orbital period, i is the inclination angle between the plane of the orbit and the plane of the sky, g is magnitude of the acceleration in the Galactic gravitational field at the location of the pulsar, λ is the direction between the gravitational field and the line of sight, Ω is the longitude of the ascending node and the geometrical factor

$$\xi(\theta) = \begin{cases} 1/\sin(\theta) & \text{for } 0 < \theta \leq \pi/2 \\ 1 & \text{for } \pi/2 < \theta < 3\pi/2 \\ -1/\sin(\theta) & \text{for } 3\pi/2 \leq \theta \leq 2\pi \end{cases} \quad (3)$$

where θ is the angle shown in Figure 1. From equation (2) it is clear that binary systems with large values of P_b^2/e , i.e., long-period circular orbit systems, are the best systems for placing limits on Δ . In order for the above equation (3) to be valid, two conditions must be met:

- The relativistic periastron advance must have completed sufficient revolutions so that its effects on e_Δ can be considered to have averaged out. Following Wex (1997), we may express this in terms of an age constraint:

$$t_{\text{system}} > 2 \times 10^8 \text{ yr} \left(\frac{P_b}{10^3 \text{ days}} \right)^{5/3} \left(\frac{M}{M_\odot} \right)^{-2/3}. \quad (4)$$

- The relativistic periastron advance must be significantly larger than the Galactic rotational velocity.

Damour & Schäfer (1991) used two pulsars that satisfied these constraints and calculated Δ using a simple Monte Carlo simulation to randomize over the unknown angles θ and Ω . Their analysis was significantly revised by Wex (1997) using an ensemble of seven binaries. Wex’s analysis is summarized in Table 1. For each system, we show the probability of Δ exceeding a given fractional level between 1 and 0.2%. In other words, the entries give the probability of the SEP being violated at the given level. In terms of the simulation, these numbers are readily calculated by simply recording the number of times Δ exceeds a given level and dividing this number by the total number of trials.

Since each binary provides an independent test of the SEP, Wex was able to place strong constraints by taking the combined probabilities for each system (i.e. the product of the individual values). The “1 – product” entry listed in the table gives the combined probability that the SEP is valid at the given fractional level. From this table, we conclude at the 90% confidence level that $\Delta < 0.004$. As smaller limits on Δ are probed it can be seen that the various binary systems begin to “drop out” of the joint test when the individual probability reaches unity. These systems do not constrain the SEP at the level $\Delta < 0.002$.

3. Two New Long-Period Binary Systems

As discussed throughout this meeting, there are many exciting results emerging from the flood of binary pulsars discovered in recent years. As far as the SEP

Pulsar	1.0%	0.5%	0.4%	0.3%	0.2%
J1455–3330	0.18	0.68	1.00	1.00	1.00
J1640+2224	0.23	0.63	0.92	1.00	1.00
J1643–1224	0.27	0.60	0.77	1.00	1.00
J1713+0747	0.08	0.18	0.23	0.39	1.00
B1953+29	0.26	0.56	1.00	1.00	1.00
J2019+2425	0.20	0.45	0.57	0.79	1.00
J2229+2643	0.17	0.76	1.00	1.00	1.00
Product	0.00	0.01	0.09	0.31	1.00
1 – Product	1.00	0.99	0.91	0.69	0.00

Table 1. Results of Wex’s (1997) analysis displayed in a slightly different form to the original version. For each binary system we display the probability that Δ exceeds a given fractional level listed at the top of each column. All probabilities are given to two decimal places.

Parameter	Unit	PSR J0407+1607	PSR J2016+1947
Spin period	ms	25.7	64.9
Characteristic age	Gyr	>1	2.5
Orbital period	days	669	635
Eccentricity		0.00095	0.00148
P_b^2/e	days ²	4.7×10^8	2.7×10^8
Companion mass	M_\odot	$0.19/\sin i$	$0.29/\sin i$

Table 2. Spin and orbital parameters for the two newly-discovered binary pulsars relevant to the SEP tests. The companion masses are based on the Keplerian orbital parameters in terms of the unknown orbital inclination i and assuming a pulsar mass of $1.35 M_\odot$.

is concerned, most of these systems do not improve upon Wex’s (1997) analysis due to either small ages, or more commonly, their low values of P_b^2/e . Two exceptions, however, are PSRs J0407+1607 and J2016+1947. Both of these binary pulsars were discovered in 430-MHz Arecibo surveys during the 1990s, the results of the survey that discovered PSR J2016+1947 have been published recently (Navarro, Anderson & Freire 2003). Only recently have these pulsars been observed long enough to accurately determinate their orbital parameters and to establish that they are old enough to satisfy equation (4) (Lorimer et al., in preparation; Freire et al., in preparation).

As can be seen from Table 2, the two new pulsars have very large values of P_b^2/e and greatly exceed the previous best system J1713+0747 (6×10^7 days²) used by Wex (1997). We note in passing that both these systems fall on the orbital period – eccentricity relation predicted by Phinney (1992).

Pulsar	0.3%	0.2%	0.09%	0.08%	0.05%
J1455–3330	1.00	1.00	1.00	1.00	1.00
J1640+2224	1.00	1.00	1.00	1.00	1.00
J1643–1224	1.00	1.00	1.00	1.00	1.00
J1713+0747	0.39	1.00	1.00	1.00	1.00
B1953+29	1.00	1.00	1.00	1.00	1.00
J2019+2425	0.79	1.00	1.00	1.00	1.00
J2229+2643	1.00	1.00	1.00	1.00	1.00
J0407+1607	0.06	0.09	0.21	0.24	0.46
J2016+1957	0.11	0.17	0.47	0.56	0.92
Product	0.00	0.02	0.10	0.13	0.42
1 – Product	1.00	0.98	0.90	0.87	0.58

Table 3. Preliminary results showing the tests of violations of the SEP for various fractional values of the parameter Δ . As in Table 1, we list the products of the individual probabilities for each system.

4. New Limits on the SEP

To demonstrate the level at which the new systems improve the constraints on Δ , in Table 3 we present some preliminary results of a new simulation which closely follows Wex’s (1997) analysis. As in this earlier analysis we exclude the long-period binary pulsars B0820+02 and B1800–27 since it is not clear whether they satisfy the age constraint. For most of the pulsars under consideration, as expected, there is no contribution to each fractional value of Δ and the test is essentially dominated by the two new pulsars for the regime $\Delta \leq 0.0002$. As a firm upper limit, we find $\Delta < 0.003$. At the 90% confidence level, we conclude that $\Delta < 0.0009$. This represents an improvement by a factor of 4.4 over the previous limits.

5. Implications

The greater self energy of neutron stars and white dwarfs over the Earth and Moon used in the solar system tests allow us to probe the term of order ϵ^2 in equation (1). Assuming from the lunar laser ranging experiments that η is negligible in this expression, and ignoring the small self energy contribution from the white dwarf, we may write

$$\Delta \simeq \epsilon_{\text{NS}}^2(\epsilon/2 + \zeta), \quad (5)$$

where the parameters ϵ and ζ are zero in GR but non-zero in alternative scalar-tensor theories (e.g., Damour & Esposito-Farese 1995). Assuming that $\epsilon_{\text{NS}} = 0.3$, our results place a 90% confidence limit on the sum $|\epsilon/2 + \zeta| < 0.001$.

References

- Damour, T., & Esposito-Farèse, G. 1995, *Phys. Rev. Lett.*, 53, 5541
Damour, T., & Schäfer, G. 1991, *Phys. Rev. Lett.*, 66, 2549
Navarro, J., Anderson, S. B., & Freire, P. C. C. 2003, *ApJ*, 594, 943
Nordtvedt, K. 1968a, *Phys. Rev.*, 169, 1014
Nordtvedt, K. 1968b, *Phys. Rev.*, 170, 1186
Phinney, E. S. 1992, *Phil. Trans. Roy. Soc. London A*, 341, 39
Wex, N. 1997, *A&A*, 317, 976
Will, C. 2001, *Living Reviews in Relativity*, 4, 4