*Binary Radio Pulsars*
*ASP Conference Series, Vol. 328, 2005*
*F. A. Rasio and I. H. Stairs*

# Gravitational Waves and the Maximum Spin Rate of Accreting Neutron Stars

Phil Arras

*Kavli Institute for Theoretical Physics*

**Abstract.** Accreting neutron stars in the weakly magnetic Low-Mass X-Ray Binaries (LMXBs) are expected to be spun up near the breakup frequency $\gtrsim 1\,\mathrm{kHz}$ in a small fraction of their age. The lack of observed systems above $\sim 600-700\,\mathrm{Hz}$ has led to the suggestion that gravitational-wave spindown torque may be setting a maximum rotation rate. If true, searches for sub-millisecond radio pulsars will not be successful, and no constraint can be placed on the equation of state of dense nuclear matter from measuring the breakup frequency. However, if gravitational waves are limiting the neutron star spin rate, the *spin period distribution* may contain information about neutron star structure and microphysics.

## 1. Introduction

A weakly magnetic neutron star accreting material with specific angular momentum $j \sim (GMR)^{1/2}$ need only accrete $\Delta M \sim 0.1 M_\odot$ to spin up to a rotation rate $\nu \sim 1\mathrm{kHz}$. The corresponding spinup time is only $t_{\rm spinup} \simeq \Delta M/\dot{M} \sim 10^8\ \mathrm{yr}\ (\dot{M}/10^{-9}\ M_\odot\,\mathrm{yr}^{-1})^{-1}$, much less than the age of a typical LMXB, implying that many accreting neutron stars in weakly magnetic LMXB should be observed rotating near the breakup frequency. Cook et al. (1994) have computed the mass shedding limit for a wide range of plausible dense matter equations of state, finding breakup frequencies $\nu_{\rm max} \sim (GM/R^3)^{1/2}/2\pi \simeq 1400 - 2500\,\mathrm{Hz}$. The naive expectation is that a large number of accreting neutron stars in LMXB should be rotating at submillisecond periods. When accretion shuts off, these systems should become submillisecond radio pulsars.

Observations of neutron star spin rates are shown in Figure 1. The dot-dashed line shows the collection of all known radio pulsars from the ATNF catalog (`www.atnf.csiro.au/research/pulsar/psrcat/`), while the dashed line shows the 20 millisecond radio pulsars found in 47 Tuc (Camilo et al. 2000). Both distributions terminate near $400 - 600$ Hz. While the selection effects for the entire sample of radio pulsars are hard to quantify, Camilo et al. (2000) have estimated the loss of sensitivity above $500 - 600\,\mathrm{Hz}$. They find a modest loss of sensitivity $\sim 20\%$, insufficient to explain the sudden drop near $\sim 500\,\mathrm{Hz}$. Although not shown in this plot, McLaughlin et al. (2004, this volume) also find few radio pulsars above $\sim 500\,\mathrm{Hz}$ in the new Arecibo drift scan survey. They *correct* the spin period distribution for the loss of sensitivity at short periods, finding the distribution is barely affected. Hence for the radio pulsars,

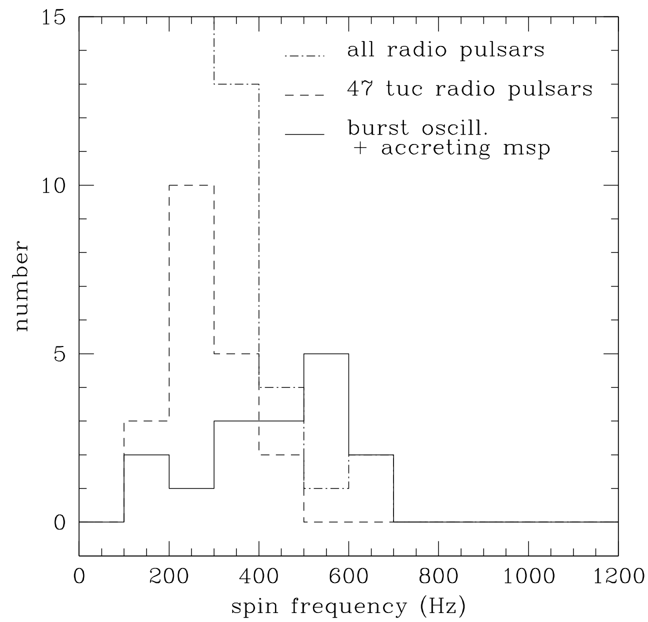


Figure 1. Neutron star spin frequency distribution. Note the lack of systems with frequency greater than $\sim 600-700$ Hz. Three populations are shown: (1) the ATNF database for all known radio pulsars (dot-dashed line), (2) the 20 radio pulsars from 47 Tuc (Camilo et al. 2000; dashed line), and (3) accreting neutron stars exhibiting persistent pulsation and/or burst oscillation (solid line).

the lack of fast pulsars may not be due to loss of sensitivity. Finally, the solid line in Figure 1 shows the spin frequencies for the five accreting millisecond pulsars, and spin frequencies determined by "burst oscillations," periodicities measured during type-1 X-ray bursts (taken from Strohmayer & Bildsten 2003). Again the distribution terminates in between $600 - 700$ Hz, although the Rossi X-ray Timing Explorer has no significant selection effects against detection at frequencies higher than 1 kHz (Chakrabarty et al. 2003).

I will now discuss two mechanisms by which gravitational waves might set an upper bound on accreting neutron star spin frequencies. I will emphasize what might be learned about neutron star structure or microphysical processes from the spin frequency distribution.

## 2. GW Torque from Crustal Quadrupoles

Bildsten (1998) first pointed out that even a tiny mass quadrupole in the neutron star crust could generate sufficient gravitational wave (GW) emission to balance the accretion torque, halting the spinup of the star. Balancing the accretion torque $\tau_{\rm acc} \simeq \dot{M}(GMR)^{1/2}$ with the GW spindown torque $N_{gw} =$

$2^{10}\pi^5 5^{-1} G Q^2 (\nu/c)^5$, where $Q$ is the mass quadrupole moment, has two immediate implications. First, the equilibrium spin frequency $\nu_{\rm eq} \propto (\dot{M}/Q^2)^{1/5}$ is insensitive to both $\dot{M}$ and $Q$, consistent with the narrow range of observed spin frequencies in LMXB's with a wide range of accretion rates. Second, for the observed range of frequencies, only $Q \sim 10^{-7,-8} MR^2$ is needed to balance the accretion torque, a tiny bump on the star. Bildsten (1998) noticed that even small horizontal temperature asymmetries could make these bumps as the temperature sensitive electron capture reactions would occur at different height as a function of latitude or longitude. The origin of the asymmetry is unknown, however Bildsten (1998) and Ushomirsky et al. (2001) argue that the asymmetric burning known to occur in type 1 X-ray bursts, or asymmetric accretion may give the required asymmetry ($\delta T(\theta,\phi)/T \sim 0.05$).

Ushomirsky et al. (2000) have performed detailed calculations of the ability of the neutron star crust to support these small mountains. For a given temperature or composition asymmetry, they compute the strain on the crust as a function of accretion rate. They find two interesting conclusions. First, the induced quadrupole moment seems to scale as $Q \propto \dot{M}^{1/2}$, canceling the dependence on accretion rate in the equilibrium spin frequency. Hence the equilibrium spin rate is expected to be quite constant over orders of magnitude in accretion rate. Second, they find the dimensionless strain in the crust becomes of order $10^{-2}$ for near-Eddington accretion rates. This is of order the breaking strain of terrestrial solids. As neutron star crusts are under high pressure, the breaking strain is not well known. Typical values assumed in the literature range from $10^{-5}$ to $10^{-2}$. *If* crustal quadrupoles are enforcing the maximum spin frequency observed for high accretion rate LMXB's, we may have a direct measurement of the breaking strain of the crust, implying it is similar to the breaking strain of terrestrial solids.

Bildsten (1998) estimates that X-ray bright LMXB's whose spin is limited by crust quadrupole GW emission are promising sources for Advanced LIGO. The observed frequency will be at twice the spin frequency of the star,

## 3. GW Torque from R-modes

Andersson (1998) and Friedman & Morsink (1998) have shown that all rotating inviscid stars will spin down due to a General Relativistic instability [1]. A certain kind of oscillation mode, called "r-modes," becomes unstable and grows exponentially due to mass-current GW emission. As angular momentum is radiated away, the mode amplitude grows and the star spins down. For accreting NS in LMXB, Bildsten (1998) and Andersson et al. (1999) first pointed out that the r-mode instability may set an upper limit on spin rates.

Whether or not this instability occurs in nature depends on if the GW mode driving rate can overcome viscous damping. Since damping typically depends on both spin frequency and core temperature, one can derive a critical spin

[1] Actually, this mechanism is likely more general. All that is needed is a sink of angular momentum. Waves in rotating stars are the crucial ingredient, not General Relativity.

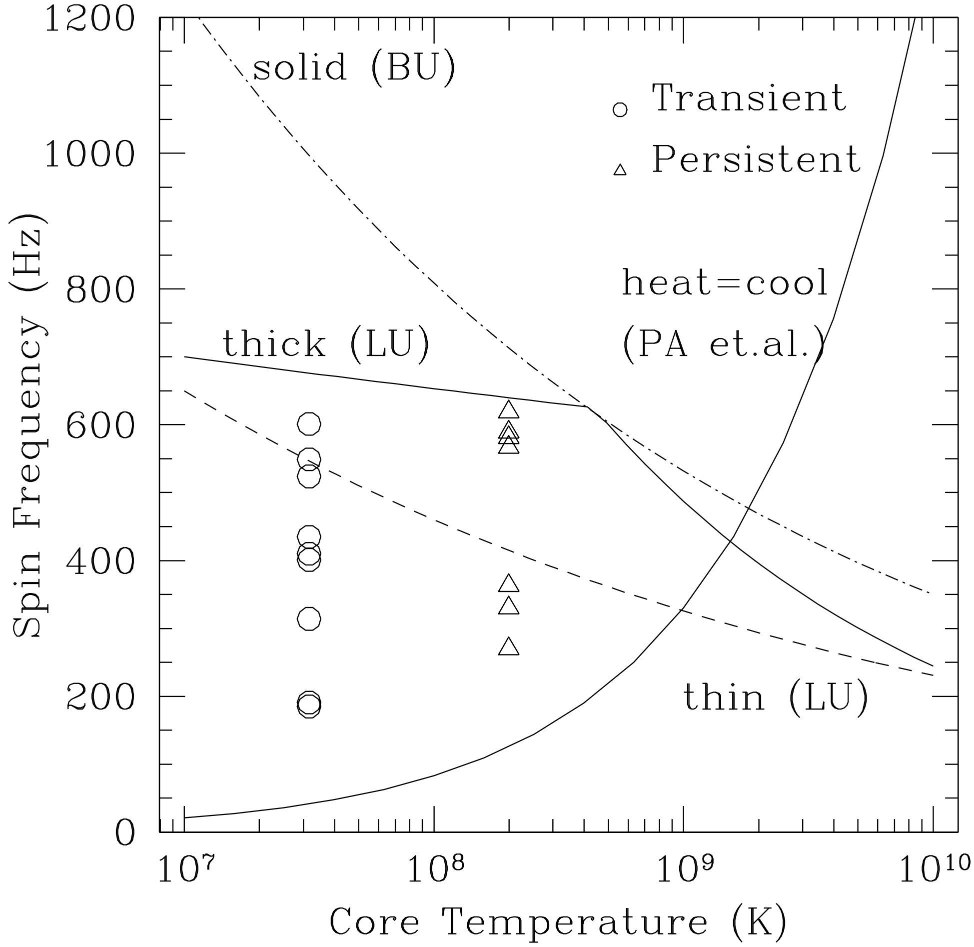


Figure 2. R-mode instability diagram for different assumptions about the crust, and normal n-p-e matter in the core. The three lines sloping down to the right give the spin frequency above which the r-mode instability operates, found by equating gravitational radiation reaction driving rate with damping rate for three different models. The dot-dashed line labeled BU is the boundary layer viscosity (Bildsten and Ushomirsky 2000), which assumes an rigid crust. The lines labeled thick (solid) and thin (dashed) use Levin and Ushomirsky's (2001) boundary layer viscosity with realistic crustal shear modulus, for a thick or thin crust.

frequency, as a function of core temperature, above which the mode is unstable. One does not expect to observe neutron stars above this critical spin frequency, as GW torque will rapidly spin them down. The point I would like to make is that the damping processes are *extremely* sensitive to the composition, structure, and microphysical processes occurring in the core and crust of the neutron star. So sensitive in fact, that one can rule out certain types of dissipation by a direct comparison with the observed spin frequencies, and inferred core temperatures.

There has been an enormous amount of work recently on viscous processes in neutron stars. Here we will look at one process in detail: viscous damping in a boundary layer at the solid crust-liquid core interface (Bildsten & Ushomirsky 2000; Levin & Ushomirsky 2001). The three lines sloping down to the right in Figure 2 show the critical spin frequency above which the mode is unstable, and hence no neutron stars should be observed. The difference between the three lines is the assumed size and rigidity of the crust; an infinitely rigid crust (dot-dashed line labeled "solid"), and thick and thin models for a crust with realistic shear modulus (solid line labeled "thick" and dashed line labeled "thin," respectively). The circles and triangles show observed spin frequencies for accreting neutron stars, measured as persistent pulsation or burst oscillations. I have somewhat artificially divided these stars into low ($\dot{M} = 10^{-11} M_{\odot}\,\mathrm{yr}^{-1}$) and high ($\dot{M} = 10^{-9} M_{\odot}\,\mathrm{yr}^{-1}$) accretion rate systems, and assigned a core temperature by balancing modified URCA neutrino emission for normal matter against a heating rate of 1 MeV per accreted baryon. In this simple case, the "thick" crust model with realistic shear modulus seems to agree best with the data, as points extend all the way up to this line, but not above it.

While the upper limit to the spin frequency is set by the process of wave damping, the expected lower limit is more complicated. In the simple model here for normal n-p-e matter, Levin (1999) has pointed out that once an accreting star is pushed above the instability line, the mode amplitude will grow exponentially and runaway heating due to viscous dissipation will move the star to the right in Figure 2. Eventually the mode amplitude will saturate by some nonlinear process and the star will spin down. Viscous heating will quickly come into equilibrium with core neutrino cooling, implying the star always spins down along the same line. In Figure 2, the line sloping down to the left uses the saturation amplitude of Arras et al. (2003), set by nonlinear wave-wave interactions, to compute the frequency as a function of temperature. Observationally, the key point is where this curve intersects the line of instability. Stars will spin down by GW emission until they exit the region of instability at this point, hence it should set the observable lower limit on the spin frequency for most systems. For our simple model, this lower limit to the frequency is at roughly 400 Hz, above roughly half the observed systems. The most obvious fix to this disagreement would be for the instability curve to drop more sharply near a core temperature of $10^9$ K. More work is needed to place strong constraints on neutron star parameters, including more recent calculations of neutron star viscosity for superfluid matter and exotic particles such as hyperons.

## 4. Conclusions

There is mounting evidence that neutron stars are not reaching their full potential, as far as rotation is concerned. After more than than 20 years of searching, no sub-millisecond pulsars have yet been found. Due to the strong dependence on rotation rate, gravitational wave spindown torque may set the observed upper bound on the rotation rate. Two mechanisms have been proposed so far: mass quadrupoles residing in the neutron star crust, or waves driven unstable by the emission of gravitational radiation. In either of these mechanisms, the spin period distribution may reveal details of neutron star structure.

**Acknowledgments.** I would like to thank my collaborators Eanna Flanagan, Sharon Morsink, Katrin Schenk, Saul Teukolsky, and Ira Wasserman. I would also like to thank Lars Bildsten, Deepto Chakrabarty and Maura Mclaughlin for useful discussions.

## References

Andersson, N. 1998, ApJ, 502, 708

Andersson, N., Kokkotas, K. D., & Stergioulas, N. 1999, ApJ, 516, 307

Arras, P., Flanagan, E. E., Morsink, S. M., Schenk, A. K., Teukolsky, S. A., & Wasserman, I. 2003, ApJ, 591, 1129

Bildsten, L. 1998, ApJ, 501, L89

Bildsten, L., & Ushomirsky, G. 2000, ApJ, 529, L33

Camilo, F., Lorimer, D.R., Freire, P., Lyne, A. G., & Manchester, R. N. 2000, ApJ, 535, 975

Chakrabarty, D., et al. 2003, Nature, 424, 42

Cook, G. B., Shapiro, S. L., & Teukolsky, S. A. 1994, ApJ, 424, 823

Friedman, J. L. & Morsink, S. M. 1998, ApJ, 502, 714

Levin, Y. 1999, ApJ, 517, 328L

Levin, Y., & Ushomirsky, G. 2001, MNRAS, 324, 917

Strohmayer, T., & Bildsten, L. 2003, in Compact Stellar X-Ray Sources, ed. W. H. G. Lewin & M. van der Klis, in press [astro-ph/0301544] (Cambridge Univ. Press, Cambridge)

Ushomirsky, G., Cutler, C., & Bildsten, L. 2000, MNRAS, 319, 902